Math 321

1.3 "Same"?
→ ① talk about the same stuff

② what you are comparing is T or F

in the exact same conditions

Def: compound proposition

if it is ① Always true → call it Tautology

② Always False → call it contradiction

ex

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

③ Contingency

Sometime True
Sometime False

ex

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Def: we call compound propositions p, q logically equiv. when $p \leftrightarrow q$ is a tautology

ex

bear	berry	#1 $\neg$bear $\wedge$ $\neg$berry	#2 $\neg$(bear $\vee$ berry)	#1 $\Leftrightarrow$ #2
T	T	F	F	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

Notation: if p and q $\boxed{\text{are}}$ logically equiv.

use $p \equiv q$

$\boxed{\text{ex}}$

$$\left.\begin{array}{l} \neg(p \vee q) \equiv \neg p \wedge \neg q \\ \neg(p \wedge q) \equiv \neg p \vee \neg q \end{array}\right] \text{DeMorgan's Laws}$$

Useful Logical Equiv. (Laws)

Identity $\quad p \wedge T \equiv p \quad , \quad p \vee F \equiv p$

Domination $\quad p \wedge F \equiv F \quad , \quad p \vee T \equiv T$

Idempotent $\quad p \wedge p \equiv p \quad , \quad p \vee p \equiv p$

double neg $\quad \neg(\neg p) \equiv p$

Commutative $\quad p \wedge q \equiv q \wedge p \quad , \quad p \vee q \equiv q \vee p$

assoc. $\quad p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

Distrib $\quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

DeMorgans $\quad \neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$$

$$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$$

$$(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$$

$$(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$$

Absorption $\quad p \vee (p \wedge q) \equiv p$

$$p \wedge (p \vee q) \equiv p$$

More equiv for $\to$ and $\leftrightarrow$

★ $p \to q \equiv \neg p \vee q$

Note: $\neg p \vee q \equiv q \vee \neg p$

q unless $\neg p$

or

★ $(p \to q) \equiv (\neg q \to \neg p)$

★ $\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv \neg(\neg p) \wedge (\neg q)$

$$\boxed{\neg(p \to q) \equiv p \wedge \neg q}$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

$$(p \leftrightarrow q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

Why?

1. show $\boxed{p \equiv q}$
   - a) use truth tables
   - b) use discussion

$$p \to q \equiv \neg p \vee q$$

2. Use them

Attend: Next class

p. 35 $\boxed{\#20}$ both table and discussion