Math 321

Q's $\boxed{\neg(p \leftrightarrow q)} \equiv (p \leftrightarrow \neg q)$

Discussion (show that both are false in exact same conditions)
or that both are True in exact same conditions)

consider $\boxed{\neg(p \leftrightarrow q)}$ when it is True

when $p \leftrightarrow q$ is false so p, q are opposite values

condition 1 p is T q is F
condition 2 p is F q is T

Consider $\boxed{(p \leftrightarrow \neg q)}$ when it is True

when p and $\neg q$ have same truth value

condition 1 p is T and $\neg q$ is T $\rightarrow$ q is F
condition 2 p is F and $\neg q$ is F $\rightarrow$ q is T

Consider: "all Math teachers are ADHD."
"some Math teachers are ADHD."
"Mark is ADHD"

$\boxed{\text{Predicate}}$

$\boxed{\text{Propositional Functions}}$

$P(x)$: "object x has predicate P"

$\boxed{\text{ex}}$ blue(x) : " x is blue"
object is from the domain (universe of discourse)
codomain is T or F

Note on U.D.

① Natural U.D. (guess what the U.D. is from the $P(x)$)

② Specified U.D. (you <u>state</u> the U.D. explicitly)

<u>n-ary propositional functions</u>:

$P(x_1, x_2, \ldots, x_n)$: "$(x_1, x_2, \ldots, x_n)$ have predicate P"

ex: $gt(x,y)$: "$x > y$"

<u>Note:</u> Propositional functions <u>are not</u> propositions

To make $P(x)$ <u>into</u> a proposition we must $\boxed{\text{bind the variable } x}$

① evaluation

ex: blue(x) : "x is blue"

blue(Mark) : "Mark is blue"

<u>Quantification</u>

② Universal "all x in the U.D. evaluate $P(x)$ to true"

③ Existential "for some (1 or more) x in the U.D. $P(x)$ is true."

Notation

Universal $\forall x\, P(x)$: "for all x in U.D, $P(x)$"

Existential $\exists x\, P(x)$: "There exists some x in U.D. such that $P(x)$"

when T? when F?

	T?	F?
$\forall x\, P(x)$	$P(x)$ is T for every x.	$P(x)$ is F for 1 or more x
$\exists x\, P(x)$	$P(x)$ is T for 1 or more	$P(x)$ is F for all x

Counter example to $\forall x P(x)$

if U.D. is finite $x_1, x_2, x_3, \ldots, x_n$

$$\forall x P(x) = P(x_1) \land P(x_2) \land \ldots \land P(x_n)$$

$$\exists x P(x) = P(x_1) \lor P(x_2) \lor \ldots \lor P(x_n)$$

DeMorgans

$$\neg \forall x P(x) \equiv \exists x\, \neg P(x)$$

$$\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$$

$r(x)$: "x is raven"

$b(x)$: "x is black"

U.D. is birds

$$\boxed{\forall q\, (r(q) \to b(q))}$$

for all birds if it is a raven, then it is black.

shorter

All ravens are black

$$\forall q\,(\boxed{r(q)} \to b(q))$$

$$\equiv \forall q\,(\neg b(q) \to \neg r(q))$$

$$\forall q\,(r(q) \to b(q)) \equiv \forall q\,(\neg r(q) \lor b(q))$$

$$\equiv \forall q\,\neg(r(q) \land \neg b(q))$$

$$\equiv \neg \exists q\,(r(q) \land \neg b(q))$$

Attend put into words

$$\forall x\,(r(x) \to b(x))$$

$$\forall x\,(r(x) \land b(x))$$

$$\exists x\,(r(x) \to b(x))$$

$$\exists x\,(r(x) \land b(x))$$