Math 321

Q's/

A	B
Knight	Knight
Knight	Knave
~~Knave~~	~~Knight~~
~~Knave~~	~~Knave~~

A: At least one of us is a Knave

Knight? → T

Knave? → F

1.6 Rules of Inference (useful tautologies)

$$\Big(\underbrace{[(p \to q) \wedge p]}_{\text{premise}} \to \underbrace{q}_{\text{conclusion}} \Big) \equiv T$$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

$$[(p \to q) \wedge p] \to q \equiv \neg[(\neg p \vee q) \wedge p] \vee q$$

$$\equiv [\neg(\neg p \vee q) \vee \neg p] \vee q$$

$$\equiv \neg(\neg p \vee q) \vee (\neg p \vee q) \equiv T$$

Argument is of the form

Ver 1   (premise $\wedge$ premise $\wedge \dots \wedge$ premise) $\rightarrow$ conclusion

Ver 2   premise
premise
$\vdots$
premise
$\therefore$ conclusion

Valid? when it is a tautology

not a tautology? Invalid

Argument form uses propositional variables

ex

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Affirm the hypothesis
$\rightarrow$ Modus ponens

ex

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Deny the conclusion
$\rightarrow$ Modus tollens

a non-tautology used as if it is (a lot)
gets its own name

$$\begin{array}{l} p \to q \\ \underline{q} \\ \therefore p \end{array}$$

fallacy of affirming the conclusion

$$\begin{array}{l} p \to q \\ \underline{\neg p} \\ \therefore \neg q \end{array}$$

fallacy of denying the hyp.

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

hypothetical syllogism

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

disjunctive syllogism

$$\begin{array}{l} \underline{p} \\ \therefore p \vee q \end{array}$$

addition

$$\begin{array}{l} \underline{p \wedge q} \\ \therefore p \end{array}$$

simplification

$$\begin{array}{l} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$$

conjunction

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore (q \vee r) \end{array}$$

resolution

<u>Quantification</u>

$$\begin{array}{l} \forall x P(x) \\ \hline \therefore P(c) \text{ for any \& the } c \end{array}$$

Universal Instantiation

$$\begin{array}{l} \exists x P(x) \\ \hline \therefore P(c) \text{ for some } c \end{array}$$

Existential Instantiation

$$\begin{array}{l} P(c) \text{ for any } c \\ \hline \therefore \forall x P(x) \end{array}$$

Universal Generalization

$$\begin{array}{l} P(c) \text{ for some } c \\ \hline \therefore \exists x P(x) \end{array}$$

<u>Note:</u>

premise
premise
⋮
premise

∴ conclusion

Must <u>know</u> premises are <u>true</u>

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ex

p.7^n

10c

All insects have 6 legs
dragonflies are insects
spiders do not have 6 legs
Spiders eat dragonflies

<u>Altered</u>
find as many valid conclusions as you can.