Math 321

Q's

(23) typo in book

$$p \wedge q$$
$$\therefore p$$

begin: $\exists x P(x) \wedge \exists x Q(x)$

typo?

but in argument use ① $\exists x P(x) \cancel{\wedge} \exists x Q(x)$

~~you can not use simplification/subtraction with $\exists$~~

$\cancel{②}$ $\exists x P(x)$

$\cancel{③}$ $P(c_1)$ some c_1

$\cancel{④}$ $\exists x Q(x)$

$\cancel{⑤}$ $Q(c_2)$ some c_2

$\cancel{⑥}$ $P(c_1) \wedge Q(c_2)$ some c_1, c_2

$\cancel{⑦}$ $\exists x (P(x) \wedge Q(x))$

Knowledge

Axiomatic Method

use reasoning to find new truths

undefined terms / Axiom : Postulates

today: $\sqrt{2}$ is irrational

NB prove $(\square \to \triangle)$ conjecture

ex 1) All A are B

2) B (you don't state the hyp)

3) $A \to B$

to prove

$(\square \to \Delta) \equiv T$

① trivial proof

② Vacuous proof

③ Direct proof

a) Assume $\square$ is true

b) use the assumption to show Δ is true

ex "the square of an even number is even"

$\equiv$ "if n is even $\to$ n^2 is even"

pf Assume n is even

$\equiv (n = 2K,\ K$ an integer$)$

$\to \quad n^2 = (2K)^2 = 4K^2 = 2(2K^2) = 2(\text{integer})$

so n^2 is even

∎

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ex show if n^2 is even $\to$ $\underline{n \text{ is even}}$

pf Assume n^2 is even

$\equiv (n^2 = 2K$, some int $K)$

stuck!

$(n = \sqrt{2K} \to \boxed{\text{show}}\ \underline{n = 2(\text{int?})}$

but, I know $(\square \to \Delta) \equiv (\neg\Delta \to \neg\square)$

contrapositive

so $(n^2 \text{ is even} \to n \text{ is even})$

$\equiv (\neg(n \text{ is even}) \to \neg(n^2 \text{ is even}))$

$\equiv (n \text{ is odd} \to n^2 \text{ is odd})$

$\boxed{\text{pf}}$ Assume n is odd $\equiv (n = 2K-1,\ K \text{ an int})$

$\Rightarrow n^2 = (2K-1)^2$

$n^2 = 4K^2 - 4K + 1 = 2(2K^2 - 2K) + 1$

$n^2 = 2(\text{int}) + 1$ which is an odd

so n^2 is odd.

<u>New facts:</u> $\boxed{n^2 \text{ even} \to n \text{ even}}$ lemma

n even $\to$ n^2 even

n odd $\to$ n^2 odd

n^2 odd $\to$ n odd

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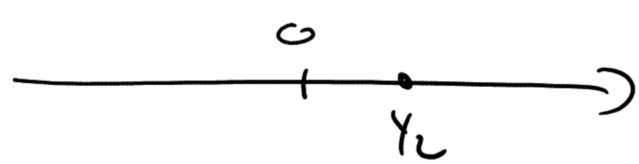
prove $\sqrt{2}$ is irrational

<u>Def:</u> x is rational means $x = \frac{a}{b}$ where a, b are integers, (a,b) have no common factors, $\wedge$ $b \neq 0$.

$\boxed{\frac{1}{2}}$, $\cancel{\frac{2}{4}}$, $\cancel{\frac{3}{6}}$



Direct Proof doesn't work. Contrapositive doesn't work.

use: want $(\square \to \Delta) \equiv T$ — to hard to

$\neg(\square \to \Delta) \equiv F$

proof by contradiction

$\boxed{(\square \wedge \neg\Delta) \equiv F}$ show this? → you have proved the conjecture

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Assume $\sqrt{2}$ is not irrational

so $\sqrt{2}$ is rational. $\sqrt{2} = \frac{a}{b}$ and a,b are ints, $b \neq 0$, (a,b) have no common factors

$\to 2 = \frac{a^2}{b^2} \to \boxed{2b^2 = a^2}$

2(int) → so $2b^2$ is even → a^2 is even

$\to \boxed{a \text{ is even}}$ (by above lemma)

so $a = 2k$, some int. k

$\to \boxed{2b^2 = a^2}$ becomes $\boxed{2b^2 = (2k)^2}$

by Algebra $2b^2 = 4k^2 \to b^2 = 2k^2$

so b^2 is even $\xrightarrow{\text{by lemma}}$ $\boxed{b \text{ is even}}$

∴ a,b have no common factor
and a,b have a common factor 2 $\Big\} \equiv F$

∴ ($\sqrt{2}$ is irrational) $\equiv T$ ∎

~~Attend~~ write up this $\uparrow$ proof