Math 321

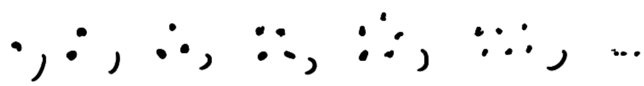
Q's/ on $\sqrt{2}$ is irrational ... (For exam)

Pf:

(A) prove lemma (if n^2 is even $\to$ n is even)
using contraposition

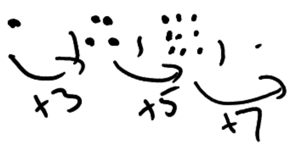
(B) prove $\sqrt{2}$ is irrational by contradiction.
(you must state definition of rational numbers)

1.7: #7 every odd is the difference of two squares.



(if I have an odd
then that odd $= (\ )^2 - (\ )^2$)

1, 4, 9, 16, 25, ..



$(a+b)^2 = a^2 + 2ab + b^2$

$(a+1)^2 = a^2 + 2a + 1$

Pf: Assume n is odd

means $n = 2k+1$, k is some integer

$\to$ $n = 2k+1+0 = 2k+1+k^2-k^2$

$\to$ $n = (k^2+2k+1) - k^2$

$\to$ $n = (k+1)^2 - (k)^2$ ∎

---

③ Prove $(p \leftrightarrow q)$ conjectures

tech #1 $\underbrace{(p \leftrightarrow q)}_{\substack{\text{|||} \\ T}} \equiv \underbrace{(p \to q)}_{\text{show this}} \land \underbrace{(q \to p)}_{\text{show this}}$

tech #2 use $(p \leftrightarrow q) \equiv T$ means what? $p \equiv q$

$p \equiv \text{step}_1 \equiv \text{step}_2 \equiv \text{step}_3 \equiv \dots \equiv q$

Problems with proofs:

① for all .....

you start doing some examples. (must do all)

② If you think the conjecture is not true always

→ find only one counter example to disprove

③ Typical mistakes:

Fallacy of assuming the conclusion

n^2 is even $\to$ n is even

pf

$n = 2k$

$\to n^2 = 4k^2 = 2(2k^2)$

$n^2 = 2K$

$(2i)^2 = 2K$

or fallacy of denying the hyp.

Conjecture $([p_1 \vee p_2 \vee \cdots \vee p_k] \to q) \equiv T$

use $[p_1 \to q] \wedge [p_2 \to q] \wedge \ldots \wedge [p_k \to q] \equiv T$

$\boxed{\text{proof by cases}}$

finite cases $\forall x P(x) \equiv \boxed{P(x_1) \wedge P(x_2) \cdots \wedge P(x_k)}$

finite $x_1, x_2, \ldots x_k$

$\boxed{\text{exhaustive proof}}$

Ints $\ldots, -2, -1, 0, 1, 2, \ldots$

neg pos

ex if $\boxed{n^2+1 < 2^n} \to \boxed{n > 4}$ for $\boxed{n \text{ a pos. int}}$

$$\equiv \left(1 \le n \le 4 \to n^2+1 \ge 2^n \right)$$

$n = 1, 2, 3, 4$

$\boxed{\text{pf}}$ $\boxed{\text{case \#1}}$ $(n=1) \to (n^2+1 \ge 2^n)$

$1^2+1 \overset{?}{\geq} 2^1$ $\boxed{\text{true}}$

do $n=2, n=3, n=4$ as well.

[Def] Conjecture $\exists x\, P(x)$

① **constructive proof** Find the c ← witness such that $P(c)$ is true.

[ex] there are two consec. integers such that one is a square and one is a cube.

[pf] Squares: $1, 4, 9, 16, 25, 36, \ldots$

Cubes $1, 8, 27, 64, \ldots$

$\boxed{8 = 2^3,\ 9 = 3^2}$

② **non-constructive** you can't find a witness...

but show $\neg \exists x\, P(x) \equiv F$

$\forall x\, \neg P(x) \equiv F$

show $c_1 \neq c_2$

show $P(c_1) \oplus P(c_2) \equiv T$
