Math 321

How to Study for Exams

What?
- Textbook Examples (+) Variation
- Homework?
- Lectures

How?
- study every day (Do not cram!)
- Coffee? ←
- energy

Q's 1.7 #9 (x is rational, y is irrational → x+y is irrational) ≡ T

Pf Contradiction?

show ¬ Conjecture is False

Show x is rational ∧ y is irrational ∧ x+y is rational

→ $x = \frac{a}{b}$ such that $\exists$: a,b are ints, (a,b) no common factors, $b \neq 0$

$x+y = \frac{c}{d}$ $\exists$: (Same as above)

Now $x+y = \frac{c}{d}$ → $y = \boxed{\frac{c}{d} + \frac{-a}{b}} = \frac{bc-ad}{bd}$

$\frac{a}{b} + y = \frac{c}{d}$

↑ we can factor and remove common factors

so y must be rational

≡ F so conj. is true

QED

Facts: rational + rational = rational

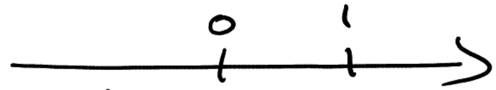
rational · rational = rational

rational + irrational = irrational

(irrational)(irrational) = ? — rational or irrational

ex $\boxed{\sqrt{2}\,\sqrt{2} = 2}$

(irrational)(irrational) = rational



Conjecture there are irrationals such that $(\text{irrational})^{\text{irrational}} = \text{rational}$ (non-constructive existence proof)

pf consider $(\sqrt{2})^{\sqrt{2}}$

<u>two cases:</u> ① $\boxed{\sqrt{2}^{\sqrt{2}}}$ is rational <u>done!</u>

or

② $\sqrt{2}^{\sqrt{2}}$ is irrational

consider $\boxed{\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}} = \sqrt{2}^2 = 2$

<u>done!</u>

Ex $\exists x\,(P(x) \wedge \forall y\,(P(y) \to y = x))$

or

$\exists x\,(P(x) \wedge \forall y\,(y \neq x \to \neg P(y)))$

<u>Uniqueness Proof</u>

Proving Stuff

① Keep in mind the start and the end

(Know what both <u>mean</u>)

{ex} x is irrational $\wedge$ y is rational $\rightarrow$ $x+y$ is irrational

② Forward / Backward reasoning

{ex} $\square \rightarrow \Delta$ (use direct proof)

Assume $\square$ $\rightsquigarrow$ work $\rightsquigarrow$ show Δ true

$\longrightarrow$ <u>forward</u> $\longleftarrow$ <u>backward</u>

③ Adapt one you did.

{ex} prove $\sqrt{3}$ is irrational

pf: <u>try contradiction</u>: (show $\sqrt{3}$ is rational $\equiv$ F)

Assume $\sqrt{3}$ is rational

$\rightarrow \sqrt{3} = \frac{a}{b}$ $\ni$; a, b are ints, $b \neq 0$, $\boxed{\text{no common factors}}$

$\rightarrow 3 = \frac{a^2}{b^2} \rightarrow 3b^2 = a^2$

New lemma: a^2 has a factor of 3 $\rightarrow$ a has a factor of 3 factor of 3 $\quad n = 3K$

¬factor of 3 $\quad \boxed{n = 3K+1}$ or $\boxed{n = 3K+2}$

Say you proved the lemma.

$3b^2 = a^2$ so a^2 has a factor of 3

by lemma $\boxed{a \text{ has a factor of } 3}$

$\rightarrow a = 3K$, K an int

$3b^2 = (3k)^2 \rightarrow 3b^2 = 9k^2 \rightarrow b^2 = 3k^2$

so b^2 has factor of 3

by lemma $\boxed{b \text{ has a factor of } 3.}$

$\rightarrow$ contradiction

$\blacksquare$