Math 321

Ch 2 Naive Set Theory

Toys (+) Rules

comparisons

Same?

ch1: propositions — $\wedge, \vee, \oplus$, $p \equiv q$, rules of inference, etc (+) apps

Ch2: Toys (+) Rules

sets — ?

Def set: unordered collection of stuff

elements or members

Notation: (1) typically use upper case as a set variable.

ex: S is a set

(2) typically use lower case as an element

ex: a is in set S.

(3) element of notation

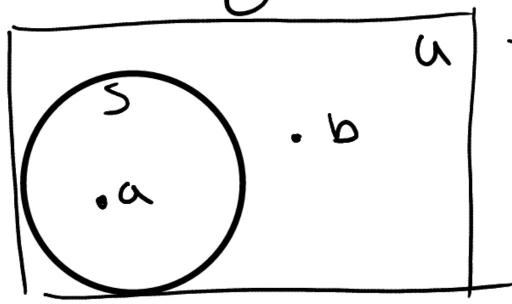
ex: $a \in S$

vs: $a \notin S$ — a is not a element of S.

$$\neg(a \in S) \equiv a \notin S$$

Representing Sets and elements.

(1) Venn Diagram



universe of discourse for all possible elements.

$a \in S$

$b \notin S$

S is a set

(2) roster or list

ex $A = \{ a, 1, \square, ☺, \}$

$B = \{ \underline{a}, \underline{\{1,2,3\}}, \underline{\square} \}$

$C = \{ a, a, a, a, z \}$

$D = \{ 1, 2, 3, \ldots, 100 \}$

$E = \{ \boxed{1}, \boxed{2}, \boxed{\{3\}} \}$

$1 \in E$

$2 \in E$

$3 \notin E$

(3) Set Builder Notation (uses logic to build the set)

$S = \{ \text{basic element type} \mid \text{Proposition function} \}$

such that

ex $S = \{ e \mid e \text{ is a integer} \wedge 1 \leq e < 6 \}$

roster → $S = \{1, 2, 3, 4, 5\}$

Sets we need to know

① U ← set of everything we are talking about

② $\emptyset$ empty set or null set

$\emptyset = \{ \}$

③ Number Sets

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ (natural or non-neg ints)

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

$\mathbb{Q} = \{ \frac{a}{b} \mid a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge b \neq 0$
$\wedge$ a,b have no common factors $\}$

$\mathbb{R}$ is all real numbers

$\mathbb{C}$ is all complex numbers

④ Singleton sets

$A = \{a\}$

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onto Rules!

→ Comparisons between sets (same?)

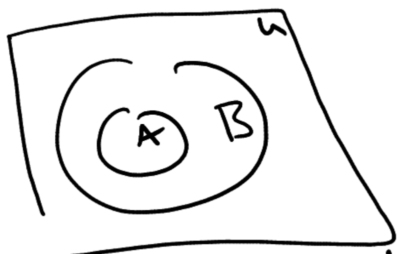
① Equality $A = B$

means $\forall c \, (c \in A \leftrightarrow c \in B)$

so $A = \{1, 1, 3, 4\}$
$B = \{1, 3, 3, 3, 4, 4\}$ $\Big\}$ $A = B$

② sub set $A \subseteq B$

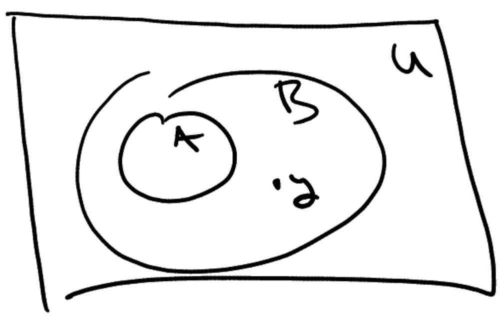
$$\forall c\,(c \in A \to c \in B)$$



③ proper subset $A \subset B$

book: $\forall x\,(x \in A \to x \in B) \land \exists x\,(x \in B \land x \notin A)$

$$\forall x\,(x \in A \to x \in B) \land \exists y\,(y \in B \land y \notin A)$$



Note: to show $A = B$

Show $\boxed{A \subseteq B} \land \boxed{B \subseteq A}$

$$(c \in A \to c \in B) \land (c \in B \to c \in A)$$

④ **Cardinality** $|S|$ = # of uniq. elements of S.

ex $|\{1, \square, \ddot{\smile}, \{\}\}| = 4$

$|\{\}| = 0$

$|\{a\}| = 1$

$|\{\{\}\}| = 1$

Def if $|S| = n$ where $n \in \mathbb{N}$
call S finite

Def if $|S|$ is not finite
we call it **infinite**

ex $|\{0, 1, 2, \ldots\}|$ = infinite

Attend

list the members of

$A = \{ x \mid x \text{ is an integer such that } x^2 = 2 \}$