Math 321 Set Theory

Sets → represent them → Venn
→ list/roster
→ set builder

→ Same? Comparison $A = B$, $A \subseteq B$, $A \subset B$

$|A|$

Ops

Special ops

① set of all subsets of a given set S

→ Power Set $\boxed{P(S)}$

Note:

$\emptyset \subseteq S$ $\forall e (e \in \emptyset \to e \in S)$

$\equiv \forall e (F \to e \in S)$

$\equiv T$

ex $S = \{1, \square, \smile\}$

subsets:

no elements $\{\ \}$

1 element $\{1\}, \{\square\}, \{\smile\}$

2 elements $\{1, \square\}, \{\square, \smile\}, \{1, \smile\}$

3 elements $\{1, \square, \smile\}$

$P(S) = \{$ put all subsets here $\}$

for this example $|S| = 3$ $|P(S)| = 8$

Computer representations of sets.

(yes, a set doesn't have order. But when we work with sets we use order to help track elements)

$S = \{ e_1, e_2, e_3, e_4, \dots, e_k \}$

I can handle subsets of S just using a bit string

$$S = \underbrace{1111\dots 1}_{k-1s}$$

b/c $e_2 \notin A$ give it 0

$A = \{ e_1, e_3 \}$ $\quad A = 10100\dots 0$

b/c $e_1 \in A$ give it 1

$\emptyset = 000 \cdots 0$

any set is a bunch of 0's or 1's.

any subset

$$|P(S)| = \left| \boxed{0 \text{ or } 1}\boxed{0 \text{ or } 1}\boxed{0 \text{ or } 1} \dots \boxed{0 \text{ or } 1} \right|$$

$$= \underbrace{2 \cdot 2 \cdot 2 \cdot \dots \cdot 2}_{|S|} = 2^{|S|}$$

$\boxed{\text{Cross Product}}$ set of all n-tuples from n-sets.

$\boxed{\text{ex}}$ $\boxed{n=2}$ $\quad A = \{0, 1, \dots, 9\}$

$B = \{a, b, c, \dots, z\}$

ordered pair. take from A then from B

$\boxed{\text{ex}}$ $\boxed{(1, c)}$

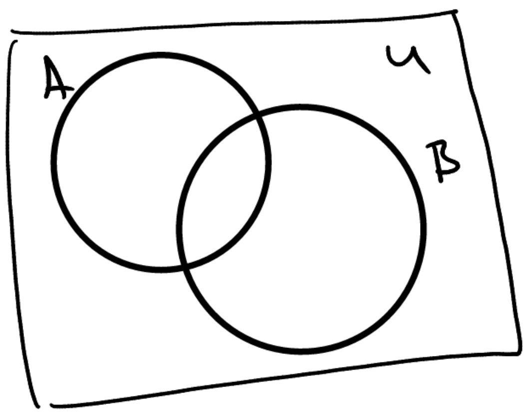
$A_1 \times A_2 = \{(a,b) \mid a \in A_1 \wedge b \in A_2\}$

$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid \forall i \; a_i \in A_i\}$

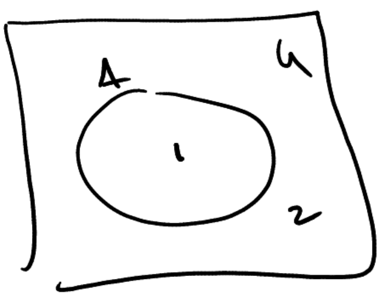
$|A_1 \times A_2 \times \cdots \times A_n| = |A_1||A_2|\cdots|A_n|$

Closed Operators

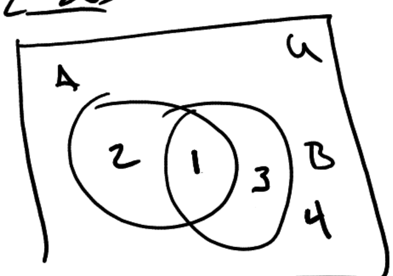
→ $\boxed{\text{Membership Tables}}$



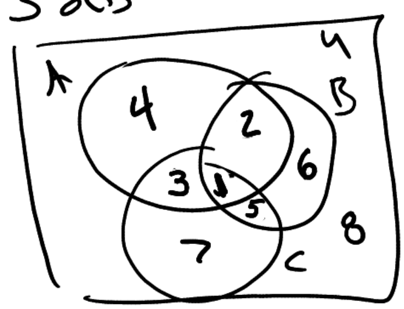
1 set



2 sets



3 sets



Membership

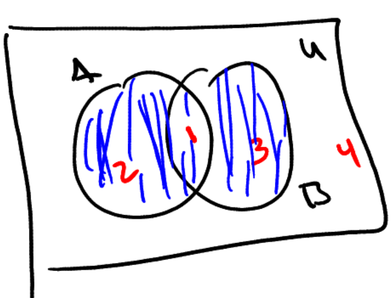
	A
area 1 →	1
area 2 →	0

A	B
1	1
1	0
0	1
0	0

A	B	C
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

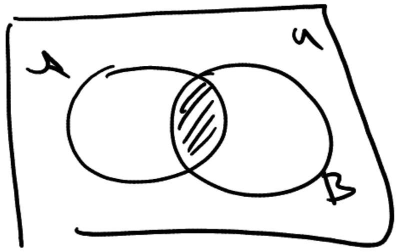
---

① $A \cup B = \{e \mid e \in A \vee e \in B\}$



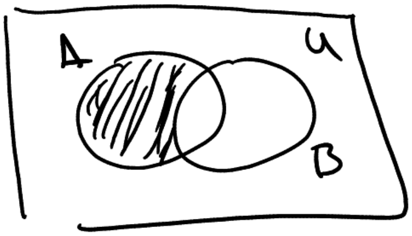
A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

② $A \cap B = \{ e \mid e \in A \wedge e \in B \}$



A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

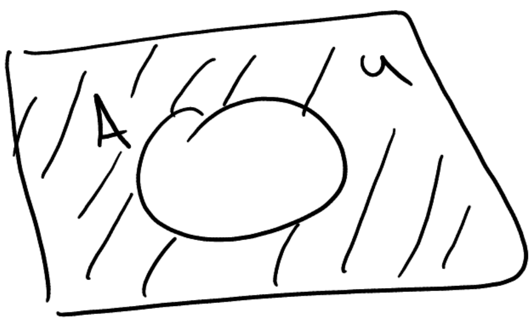
③ $A - B = \{ e \mid e \in A \wedge e \notin B \}$



A	B	$A - B$
1	1	0
1	0	1
0	1	0
0	0	0

④ $U - A = \{ e \mid \boxed{e \in U} \wedge e \notin A \}$

$= \{ e \mid \neg (e \in A) \} = \overline{A}$



$\overline{(A \cap B)}$

A	B	$A \cap B$	$\overline{A \cap B}$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

$\overline{A} \cup \overline{B}$

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cup \overline{B}$
1	1	0	0	0
1	0	0	1	1
0	1	1	0	1
0	0	1	1	1

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\boxed{\text{But}}$

$$\overline{A \cap B} = \{e \mid e \notin (A \cap B)\}$$
$$= \{e \mid \neg(e \in A \cap B)\}$$
$$= \{e \mid \neg(\underline{\underline{e \in A}} \wedge \underline{\underline{e \in B}})\}$$
$$= \{e \mid e \notin A \vee e \notin B\}$$
$$= \{e \mid e \in \overline{A} \vee e \in \overline{B}\}$$
$$= \overline{A} \cup \overline{B}$$

<u>Set Identities / laws</u> (p. 130)

$$A \cap U = A \qquad A \cap \emptyset = \emptyset$$
$$A \cup \emptyset = A \qquad A \cup U = U$$