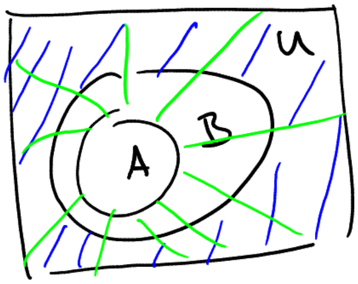
Math 321

Q's / (31)



thm: $A \subseteq B$ iff $\overline{B} \subseteq \overline{A}$

left $\Longleftrightarrow$ right

Pf tech #1 case 1 $(A \subseteq B) \to (\overline{B} \subseteq \overline{A})$

case 2 $(\overline{B} \subseteq \overline{A}) \to (A \subseteq B)$

✓ tech #2 $(A \subseteq B) \equiv \sim \equiv \sim \equiv \cdots \equiv (\overline{B} \subseteq \overline{A})$

Note: stuff to know $(A \subseteq B)$

mean $\forall e (e \in A \to e \in B)$

$(\overline{B} \subseteq \overline{A})$

mean $\forall e ((e \in \overline{B}) \to e \in \overline{A})$

$\equiv \forall e (\neg (e \in B) \to \neg (e \in A))$

Pf $(A \subseteq B) \overset{\text{by def}}{\equiv} (\forall e (e \in A \to e \in B))$

by contrapositive

$\equiv (\forall e (\neg (e \in B) \to \neg (e \in A)))$

$\equiv (\forall e (e \in \overline{B} \to e \in \overline{A}))$

$\equiv (\overline{B} \subseteq \overline{A})$

$\boxed{2.3}$ Functions (Application & Sets)

① Relationships.

consider $A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \ldots, a_n) \mid \forall i\, a_i \in A_i \}$

n-ary relation : any subset of those possible n-tuples.

(finding the rule to represent an n-ary relation is really hard)

So ... Simplify

② Binary Relation

$$A \times B = \{ \underbrace{(a,b)}_{\text{ordered pairs}} \mid a \in A \wedge b \in B \}$$

any subset of $A \times B$ is a binary relationship..

A : domain

B : codomain

→ easier to find a "rule" to represent the binary relation.

③ $\boxed{\text{Functions}}$ set of ordered pairs

such that exactly one element of B is assigned to each element of A.

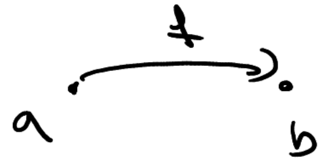
Notations for functions

$a \in A \quad b \in B$

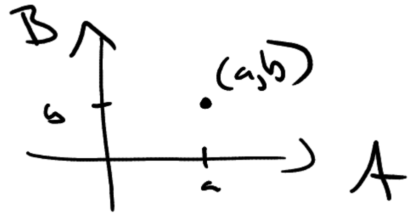
A: domain
B: codomain
a: preimage
b: image

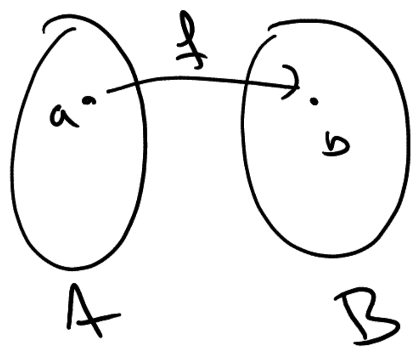
$$f(a) = b$$

$$f : A \to B$$



$$(a,b) \in f$$





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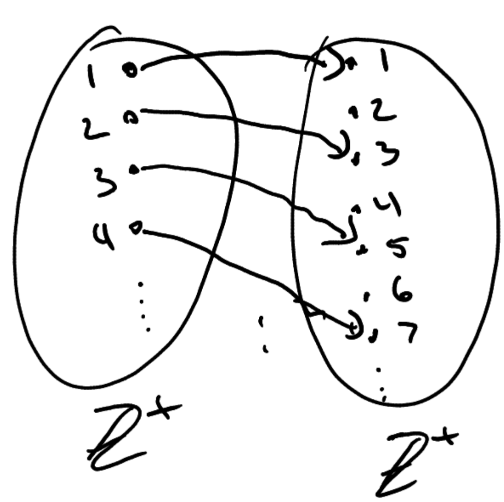
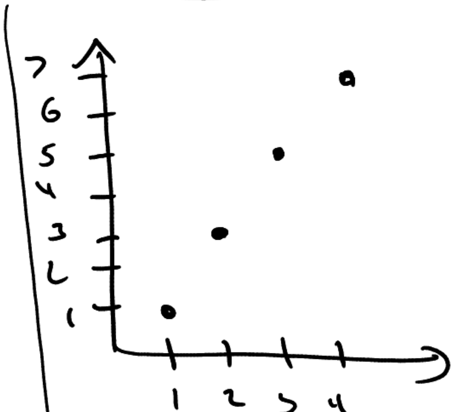
ex $\quad f : \mathbb{Z}^+ \to \mathbb{Z}^+$

$$f(n) = 2n - 1$$



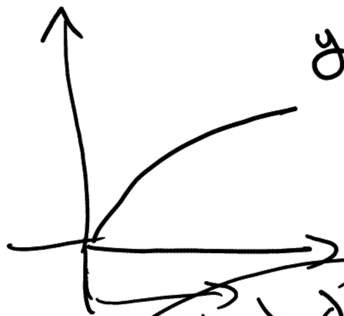
Graphs



$$f = \{(1,1), (2,3), (3,5), \ldots\}$$

Domains: all elements of domain must map.
(to be a function)

[ex] $y = \sqrt{x}$ $\quad f : \boxed{\mathbb{R}} \to \mathbb{R}$



Natural Domains

restrict domain to $x \geq 0$

→ **Partial Functions** $\quad f : A \to B$

domain A has some restrictions on use.

[ex] $f(x) = \dfrac{1}{x^2 - 4}$ ← partial function

b/c we need to not use $x = 2$
$x = -2$

$f(x) = \sqrt{x+2}$ ← partial function b/c $\sqrt{\square}$
must be non-neg.

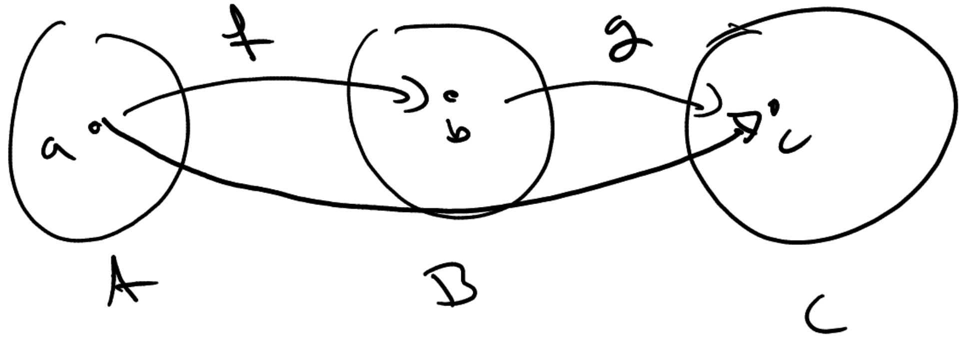
Ops on functions $\quad f : A \to B$
$\quad g : A \to B$

[ex] $f : A \to \mathbb{R}$, $\quad g : A \to \boxed{\mathbb{R}}$

$(f \pm g)(x) = f(x) \pm g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$

Composition



$$f(a) = b \qquad g(b) = c$$

$$\Rightarrow \quad g(f(a)) = c$$

$$(g \circ f)(a) = c$$

Composition

ex) $g(x) = x^3 \qquad f(x) = x - 1$

$$(f \circ g)(3) = f(g(3)) = f(27) = 26$$

$$(g \circ f)(3) = g(f(3)) = g(2) = 8$$

Identities / Inverses.

① $f + g$

a) Identity $\quad (f + I)(x) = f(x)$

$I(x) = 0$

b) Inverse $\quad (f + -f)(x) = 0$

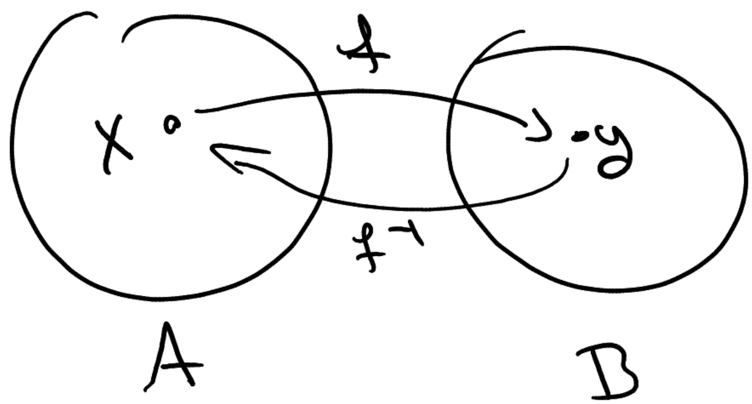
② $f \circ g$

a) Identity $(f \circ I)(x) = f(x)$

$f(I(x)) =$

$\underline{\underline{I(x) = x}}$

b) <u>Inverse</u> $(f \circ f^{-1})(x) = x$



f^{-1} <u>exist?</u> <u>show it?</u> <u>find it?</u>