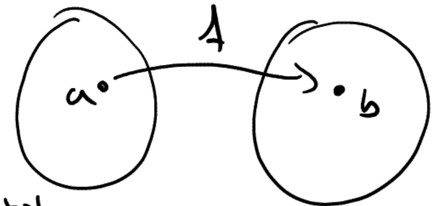
Math 321

Functions — types

① one-to-one function



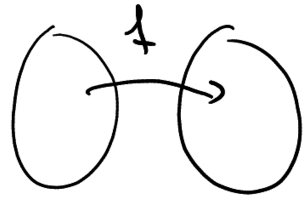
$b = f(a)$ no other pre-image goes to b

$$\forall x_1 \forall x_2 \left(x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2) \right)$$

$$\equiv \forall x_1 \forall x_2 \left(f(x_1) = f(x_2) \rightarrow x_1 = x_2 \right)$$

② onto function

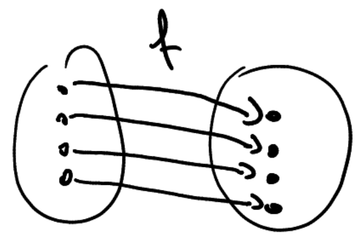
$$\forall y \, \exists x \, (f(x) = y)$$



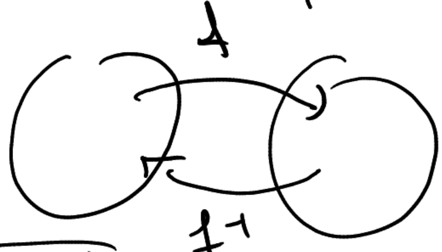
everyone in codomain has a pre-image

③ one-to-one correspondence (bijection)

f is ① one-to-one
<u>and</u>
② onto



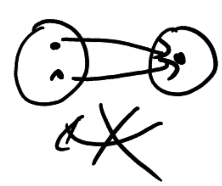
Inverse under composition



f^{-1} exists if
f is one-to-one and onto

functions that can not have inverses..

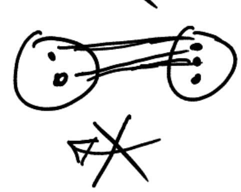
[ex]



not one-to-one

or

[ex]



not onto

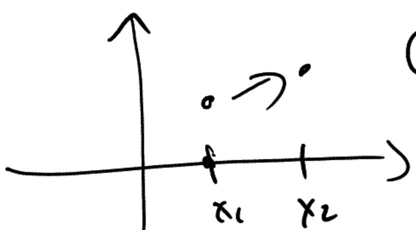
Check if you found f^{-1}

$$(f \circ f^{-1})(x) = x$$
$$(f^{-1} \circ f)(x) = x$$

Inc / Dec Functions

Inc.

① $\forall x_1 \forall x_2 \left(x_1 < x_2 \rightarrow f(x_1) \leq f(x_2) \right)$



② $\forall x_1 \forall x_2 \left(x_1 < x_2 \rightarrow f(x_1) < f(x_2) \right)$

strictly inc.

dec.

③ $\forall x_1 \forall x_2 \left(x_1 < x_2 \rightarrow f(x_1) \geq f(x_2) \right)$

④ $\forall x_1 \forall x_2 \left(x_1 < x_2 \rightarrow f(x_1) > f(x_2) \right)$

strictly dec.

Functions to Know

$f : \mathbb{R} \rightarrow \mathbb{Z}$

① floor $\quad \lfloor x \rfloor = \begin{cases} x & \text{if } x \in \mathbb{Z} \\ \text{1st integer below } x & \text{if } x \notin \mathbb{Z} \end{cases}$

ex

$\lfloor -5 \rfloor = -5$

$\lfloor 1.2 \rfloor = 1$

$\lfloor -1.001 \rfloor = -2$

② ceiling $\quad \lceil x \rceil = \begin{cases} x & \text{if } x \in \mathbb{Z} \\ \text{1st int. above } x & \text{if } x \notin \mathbb{Z} \end{cases}$

ex $\lceil -5 \rceil = -5, \quad \lceil -1.0001 \rceil = -1$

$f: \mathbb{R} \to (x \geq 0)$

↖ non-neg reals

① $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$f: \mathbb{N} \to \mathbb{Z}^+$

① $n!$

$0! = 1$

$1! = 1$

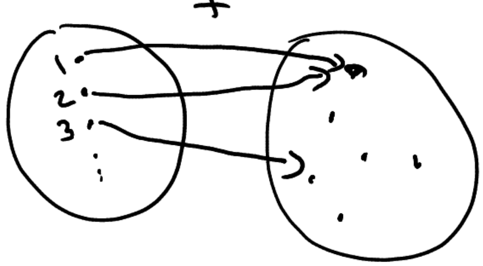
$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

Seq's / Summations

Sequence is just a function from some subset of the integers (usually $\{0, 1, 2, \ldots\}$ or $\{1, 2, 3, \ldots\}$) to a set S

f



S

$f(1)$, $f(2)$, $f(3)$, $f(4)$, $\vdots$

$(1, f(1))$, $(2, f(2))$, $\vdots$

<u>Notation:</u> $(1, f(1)) \rightsquigarrow$ write f_1

$(2, f(2)) \rightsquigarrow$ write f_2

<u>Sequence:</u> $\boxed{f_1, f_2, f_3, f_4, \ldots}$

Notation: $\{a_n\}$ = function
rule to make the seq.

ex
① geometric sequence $\quad a_n = a r^n$
$n = 0, 1, 2, \ldots$

ex $\{3(-2)^n\}$ $\quad n = 0, 1, 2, \ldots$ rule to seq / rule to seq
seq $= 3, -6, 12, -24, \ldots$

② Arithmetic Seq $\quad a_n = a + d \cdot n \quad n = 0, 1, 2, \ldots$

ex $\{1 + 3n\}$ $\quad n = 0, 1, 2, \ldots$
seq $= 1, 4, 7, 10, \ldots$

③ $a_n = n^2 \quad n = 1, 2, 3, \ldots$
seq $= 1, 4, 9, 16, \ldots$

④ $a_n = n^3 \quad n = 1, 2, 3, \ldots$
seq $= 1, 8, 27, \ldots$

⑤ $a_n = 2^n \quad n = 0, 1, 2, \ldots$
seq $= 1, 2, 4, 8, \ldots$

⑥ $a_n = n! \quad n = 0, 1, 2, 3, \ldots$
seq $= 1, 1, 2, 6, 24, \ldots$

open form of a seq (vs) closed form of a seq

recurrence relation or Inductive

(1) Basis $\boxed{a_0 = 1}$

(2) Recurrence Relation

(New values) = (Rule on old values)

$$\boxed{a_n = 2 \cdot a_{n-1} \quad n = 1, 2, 3, \ldots}$$

a_0 a_1 a_2

1, 2, 4, 8, 16, ..

function

$a_n = 2^n$

$n = 0, 1, 2, \ldots$

1, 2, 4, 8, 16, ..

---

ex

$f_0 = 0$, $f_1 = 1$ (Basis)

$f_n = f_{n-1} + f_{n-2} \quad n = 2, 3, 4, \ldots$

seq: f_0, f_1, f_2, f_3, f_4, ..

0, 1, 1, 2, 3, 5, 8, ..

---

open → closed ?

(1) guess?

(2) Forward vs backward Substitution

ex $a_0 = 1$

$a_n = 2a_{n-1} \quad n = 1, 2, 3, \ldots$

Forward Substitution.

$a_{\boxed{0}} = 1$, $a_1 = 2a_0 = 2 \cdot 1$, $a_2 = 2a_1 = 2 \cdot 2 = 2^2$

$a_3 = 2a_2 = 2(2^2) = 2^3$

$a_4 = 2a_3 = 2(2^3) = 2^4$

$\vdots$

$$\boxed{a_n = 2^n}$$

Backward Substitution

$a_n = 2a_{n-1}$ but $a_{n-1} = 2a_{n-2}$

$a_n = 2(2a_{n-2}) = 2^2 a_{n-2}$

$a_n = 2^2(2a_{n-3}) = 2^3 a_{n-3}$

$a_n = 2^3(2a_{n-4}) = 2^{\boxed{4}} a_{n-\boxed{4}}$

$\vdots$

$a_n = 2^n a_{n-n} = 2^n a_0 = 2^n(1) = 2^n$

$$\boxed{a_n = 2^n}$$

Summation given seq $a_n, a_{n+1}, a_{n+2}, \ldots, a_m$

Summation is $\boxed{a_n + a_{n+1} + a_{n+2} + \cdots + a_m}$

Notation: $\sum_{i=n}^{m} a_i$

ex

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1 + 4 + 9 + \cdots + n^2 = \boxed{}$$

Find this

ex

$$1 + 2 + 3 + 4 + 5 + 6$$

(Attend)