Math 321

Seq: $\{a_n\}$ $n = 0, 1, 2, \dots$

→ $a_0, a_1, a_2, a_3, \dots$

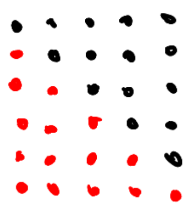
Sum $\displaystyle\sum_{i=n}^{m} a_i = a_n + a_{n+1} + \dots + a_m$

ex $\displaystyle\sum_{i=3}^{7} (2+3i) = \underset{n=3}{11} + \underset{n=4}{14} + \underset{n=5}{17} + \underset{n=6}{20} + \underset{n=7}{23} = ?$

ex
$$
\begin{array}{l}
1 + 2 + 3 + \dots + 100 \\
100 + 99 + 98 + \dots + 1 \\
\hline
101 + 101 + 101 + \dots + 101 = 100(101)
\end{array}
$$

$= \dfrac{100(101)}{2} = 50(101) = \boxed{5050}$

$1+2+3+4+5$



→ Area $= 5(6)$

area of Δ number $= \dfrac{5(6)}{2}$

$$\sum_{i=1}^{n} i = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$$

→ telescoping series $\displaystyle\sum_{i=1}^{n} (a_{i+1} - a_i)$

$$= \underset{i=1}{(\cancel{a_2} - a_1)} + \underset{i=2}{(\cancel{a_3} - \cancel{a_2})} + \underset{i=3}{(\cancel{a_4} - \cancel{a_3})} + \dots + \underset{i=n-1}{(\cancel{a_n} - \cancel{a_{n-1}})} + \underset{i=n}{(a_{n+1} - \cancel{a_n})}$$

$$\hookrightarrow \; = a_{n+1} - a_1$$

$$(i+1)^2 - (i)^2 = i^2 + 2i + 1 - i^2 = 2i+1$$

$$(i+1)^2 - (i)^2 = 2i+1$$

$$(i+1)^3 - (i)^3 = i^3 + 3i^2 + 3i + 1 - i^3 = 3i^2 + 3i + 1$$

$$(i+1)^3 - (i)^3 = 3i^2 + 3i + 1$$

$$\sum_{i=1}^{n} (i+1)^3 - (i)^3 = \sum_{i=1}^{n} \left(3i^2 + 3i + 1\right)$$

$\downarrow$ tele.

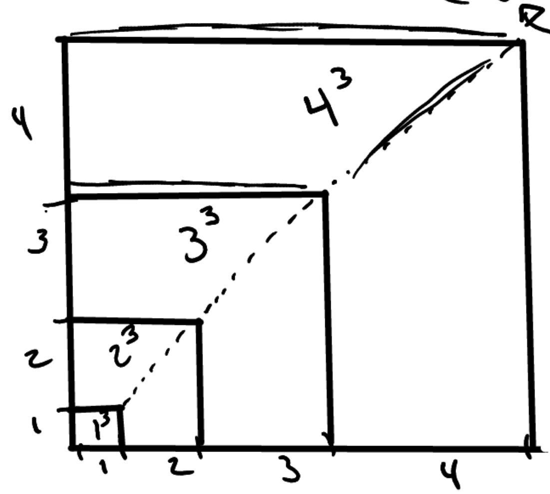
$$(n+1)^3 - (1)^3 = 3\left(\sum_{i=1}^{n} i^2\right) + 3\underbrace{\left(\sum_{i=1}^{n} i\right)}_{\frac{n(n+1)}{2}} + \underbrace{\sum_{i=1}^{n} 1}_{n}$$

$\rightarrow$ algebra

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

ex $\quad 1^3 + 2^3 + 3^3 + 4^3 = \left(\frac{4(5)}{2}\right)^2 \leftarrow 1+2+3+4$



$$\sum_{i=0}^{n} ar^i = a + ar + ar^2 + ar^3 + \cdots + ar^n = S$$

$$S = a + ar + ar^2 + \cdots + ar^n$$

$$Sr = ar + ar^2 + ar^3 + \cdots + ar^n + ar^{n+1}$$

$$(Sr - S) = ar^{n+1} - a$$

$$S(r-1) = ar^{n+1} - a$$

$$S = a\left(\frac{r^{n+1}-1}{r-1}\right) \quad (r \neq 1)$$

$$\boxed{\sum_{i=0}^{n} ar^i = \begin{cases} a\left(\frac{r^{n+1}-1}{r-1}\right) & \text{if } r \neq 1 \\ a(n+1) & \text{if } r = 1 \end{cases}}$$

we can use this tech to show $1 = 0.\bar{9}$

$$S = .99999\ldots = 9\left(\tfrac{1}{10}\right) + \cancel{9\left(\tfrac{1}{100}\right)} + \cancel{9\left(\tfrac{1}{1000}\right)} + \cdots$$

$$10S = 9.999\ldots = 9 + \cancel{9\left(\tfrac{1}{10}\right)} + \cancel{9\left(\tfrac{1}{100}\right)} + \cancel{\cdots}$$

$$(10S - S) = 9 \rightarrow 9S = 9 \quad \boxed{S = 1}$$

Note: all term decimals have $\boxed{\text{two}}$ forms

$\boxed{\text{ex}}$ $0.125 = 0.1249999\ldots$

$0.3 = 0.29999\ldots$

Cardinality

$$|S| = n \in \mathbb{N} = \{0, 1, 2, \ldots\}$$

→ S is called <u>finite</u>.

→ if not we call S <u>infinite</u>.

on <u>same cardinality</u>

$$\left.\begin{array}{l} |A| = |\{1,2,3\}| = 3 \\ |B| = |\{a,b,c\}| = 3 \end{array}\right\} \text{ say } |A| = |B|$$

Def $|A| = |B|$ <u>iff</u> there is a bijection from A to B.

Def

① if there is a one-to-one function we can say $|A| \le |B|$

② if one-to-one and not onto $|A| \neq |B|$ we say $|A| < |B|$

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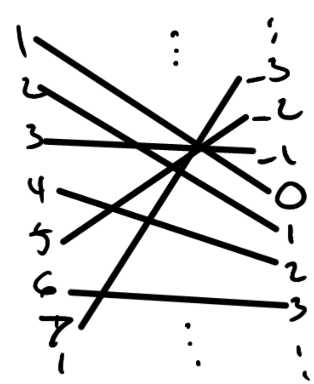
ex



So $|\mathbb{Z}^+| = |\mathbb{Z}|$

<u>Countable</u>

$|\mathbb{Z}^+| = \aleph_0$

Def a) any set whose $\boxed{|S| = \aleph_0}$ is called countable.

b) or $|S| = n \in \mathbb{N}$

find a bijection from $\mathbb{Z}^+$ to S!

Hilberts Grand Hotel

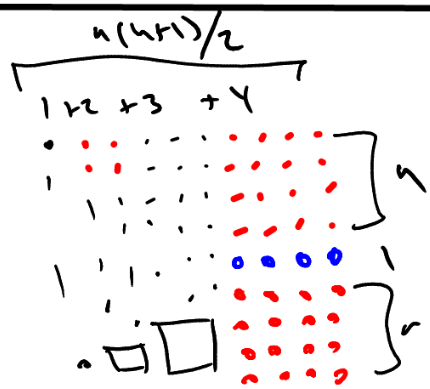
rooms: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots$

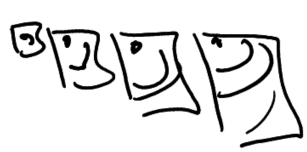
groups:

$$\begin{array}{ccccc} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

$|\mathbb{Q}| = |\mathbb{Z}^+|$

Attend: read and write up textbook proof for $|\mathbb{Q}^+|$





$$\left(\frac{n(n+1)}{2}\right)\left(\frac{2n+1}{3}\right) = \frac{n(n+1)(2n+1)}{6}$$