Math 321

Applications of countable vs uncountable

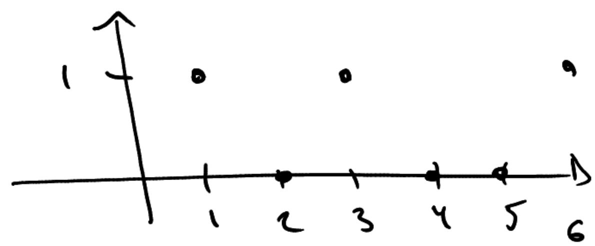
→ Reals from 0 to 1 are uncountable

$$r_i = 0.d_{i1}d_{i2}d_{i3}\ldots$$

ex $\boxed{r = 0.1010010001\ldots}$ → "acts like" a function from $\mathbb{Z}^+ \to \{0,1,2,\ldots 9\}$

→ $f(n)$ = n^{th} decimal of our number

$f(3) = 1$
$f(4) = 0$
$f(5) = 0$
$f(6) = 1$
$\vdots$



→ If every real from 0 to 1 has a function "version" of it

→ $f : \mathbb{Z}^+ \to \{0,1,2,\ldots,9\}$ are <u>uncountable infinite</u>

Programming uses <u>strings</u> → $\boxed{\text{a finite set}}$ ← <u>countable</u>

program $= s_1 s_2 s_3 s_4 \ldots$ → $\boxed{\text{countable infinite}}$

s_1 from a $\boxed{\text{finite set}}$

Function has a program → computable

Function does not have a program → uncomputable b/c all functions are uncountable but programs are countable.

$\rightarrow$ uncountably infinite group of uncomputable functions.

Needed Tool: Matrices

Def: Matrix: rect. array of real numbers

ex: $A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow A$ is 2×3

dimensions: row x col

Notation: $A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

Ops: ① $A = B$

$\rightarrow$ both are same size $(m \times n)$

$\rightarrow$ $a_{ij} = b_{ij}$ for all i, j

② $A + B = [a_{ij} + b_{ij}]$

$\rightarrow$ both are $m \times n$

③ $\underset{m \times k}{A} \; \underset{k \times n}{B} = \underset{m \times n}{C}$

where $c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{ik} b_{kj}$

$$\begin{array}{c} \\ i^{th} \text{ row} \end{array} \begin{bmatrix} \boxed{a_{i1} \; a_{i2} \cdots a_{ik}} \\ \\ \\ \end{bmatrix} \begin{array}{c} j^{th} \text{ col} \\ \begin{bmatrix} & \boxed{\begin{matrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{kj} \end{matrix}} & \end{bmatrix} \end{array} = \begin{bmatrix} & c_{ij} & \\ & & \\ & & \end{bmatrix}$$

(4) $A^T = \{a_{ji}\}$ swap row/col.

Note: $A^T = A \rightarrow$ call A symmetric.

Special Matrices:

all zeros

$O = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & - & - & 0 \end{bmatrix}$ Additive Identity

$I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$ Multiplicative Identity

Additive Inverse: $A + (-A) = O$

Multiplicative Inverse $A A^{-1} = I$

$A^{-1} A = I$

(5) $A^r = A^{r-1} \cdot A$

define $A^0 = I$

A must be $n \times n$

For this class you should be able to..

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 0 & 3 \end{bmatrix}$

$A + A = ?$ $A^5 = ?$

$AB = \begin{bmatrix} 5 & 4 & -1 & 6 \\ 11 & 10 & -3 & 12 \end{bmatrix}$

type: Zero-one Matrices : rect array of bits

$b \in \{0, 1\}$ is called a bit

ex: $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

ops: ① equality $\quad A = B$ → same size
→ same stuff in same spots

② join $\quad A \vee B = [a_{ij} \vee b_{ij}]$

③ meet $\quad A \wedge B = [a_{ij} \wedge b_{ij}]$

+ xor $\quad A \oplus B = [a_{ij} \oplus b_{ij}]$

④ Boolean Product $\quad \underset{m \times k}{A} \odot \underset{k \times n}{B} = C$

$$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \dots \vee (a_{ik} \wedge b_{kj})$$

⑤ $\quad A^{[n]} = A^{[n-1]} \odot A$

$A^{[0]} = I$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A \wedge B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad A \vee B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$CB = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$a,b,c \to a,b,c$

$$R = \{(a,a), (a,c), (b,a), (c,a), (c,c)\}$$

why zero-one matrices

$$M_R = \begin{array}{c|c} & \begin{array}{ccc} a & b & c \end{array} \\ \hline \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

A

For Monday:

A

Find ① $A + A$
② $A \cdot A$
③ $A \odot A$
④ $A^3 + A$