Math 321

Q's / 2.6 #1a

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

just $AA^{-1} = \sim \text{work} \sim = I$

$A^{-1}A = \sim \text{work} \sim = I$

$$\begin{bmatrix} a & b \\ c & a \end{bmatrix} \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{ad}{ad-bc} - \frac{bc}{ad-bc} & \\ & \text{etc} \end{bmatrix}$$

2.5 #a

$r_1 \quad r_2 \quad r_3 \quad \ldots$ $\exists$ Hotel

people: $b_{0,1}$ $b_{0,2}$ $b_{0,3}$, ...

New people: $b_{1,1}$ $b_{1,2}$ $b_{1,3}$, ...

$b_{2,1}$ $b_{2,2}$ $b_{2,3}$, ...

$d=1$ $d=2$ $d=3$

countable (for infinite sets) means: bijection (function) from $\mathbb{Z}^+$ to our set bijection: $\mathbb{Z}^+ \xrightarrow{f}$ People

$$
\begin{array}{lcll}
1 & \longleftrightarrow & b_{0,1} & \} \; d=1 \\
2 & \longleftrightarrow & b_{1,1} & \multirow{2}{*}{$\}\; d=2$} \\
3 & \longleftrightarrow & b_{0,2} & \\
4 & \longleftrightarrow & b_{2,1} & \\
5 & \longleftrightarrow & b_{1,2} & \; d=3 \\
6 & \longleftrightarrow & b_{0,3} & \\
7 & & \vdots & \\
8 & & &
\end{array}
$$

explain the diagonals and why no $b_{m,n}$ is missed.

$$\boxed{d = m+n}$$

$\# \longleftrightarrow b_{m,n}$ $\qquad b_{100,102}$

$$f(m,n) = \#$$

2.4 17d $\quad a_n = a_{n-1} + 2n + 3, \quad a_0 = 4$

$$\{a_n\} = \underset{n=0}{4},\ \underset{n=1}{\overset{4+2(1)+3}{9}},\ \underset{n=2}{16},\ \underset{n=3}{25},\ \ldots$$

tech #1 $\quad \boxed{\text{Guess}}$

$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
4	9	16	25	36
2^2	3^2	4^2	5^2	6^2

$$\boxed{a_n = (n+2)^2} \qquad n = 0, 1, 2, \ldots$$

check:
$$a_n = a_{n-1} + 2n + 3$$

$$(n+2)^2 \overset{?}{=} (n+1)^2 + 2n + 3$$

$$n^2 + 4n + 4 \overset{?}{=} n^2 + 2n + 1 + 2n + 3 = n^2 + 4n + 4$$

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tech #2 $\quad$ iteration.
$$a_n = a_{n-1} + 2n + 3$$
$$a_0 = 4$$

$a_0 = 4$

$a_1 = a_0 + 2(1) + 3 = 4 + 2(1) + 3$

$a_2 = a_1 + 2(2) + 3 = 4 + 2(1) + 3 + 2(2) + 3$

$4 + 2(2) + 2(1) + 2(3)$

$a_3 = a_2 + 2(3) + 3 = 4 + 2(2) + 2(1) + 2(3)$

$+ 2(3) + 3$

$= 4 + \underline{2(3) + 2(2) + 2(1)} + 3(3)$

$a_3 = 4 + 2(1+2+3) + 3(3)$

$a_n = 4 + 2(\underbrace{1+2+3+\dots+n}_{\frac{n(n+1)}{2}}) + n(3)$

$a_n = 4 + n(n+1) + 3n = n^2 + 4n + 4$

$\boxed{a_n = (n+2)^2}$

$a_n = a_{n-1} + 2(n) + 3$

$a_n = (a_{n-2} + 2(n-1) + 3) + 2(n) + 3$

$a_n = (a_{n-3} + 2(n-2) + 3) + (2(n-1) + 3) + 2(n) + 3$

$\vdots$

$a_n = a_0 + 2(1+2+\dots+n) + n(3)$

e/z

Exam 2

11 probs

2.1 Sets (representations and crossprod and P(s))

2 probs

1. representation.
2. Find cross product
   Find $P(S)$

ex $A = \{1,2\}$ $B = \{a,b\}$

$$P(A \times B) = ?$$

2.2 Set ops. (7 probs)

1. set ops w/ membership tables.

ex give the membership table for $\overline{A \cap B \cup C}$
(+) Venn diagram

2. Set ops by set builder notation..

ex $\overline{A \cup B} = \overline{A} \cap \overline{B}$ by set builder.

$$\overline{A \cup B} = \{e \mid \neg(e \in A \cup B)\}$$
$$= \{e \mid \neg(e \in A \vee e \in B)\}$$
$$= \text{etc}$$

2.3 Functions (2 probs)

1. p. 154 #34 or #35 ✓
2. Create functions that are ..
   one-to-one?
   onto?
   both?
   neither?

2.4 **Seqs Sums** (2 probs)

① several part seq.

rule → seq

seq → rule

② Sum (Know the sums from the section)

2.5 Countability (2 probs)

① $\mathbb{Q}$ is countable ✓

② $\mathbb{R}$ is uncountable ✓

2.6 Matrices (1 prob)

① OPS