# Math 321

Q's ~~/~~ 4.3 #11

Show: $\log_2 3$ is irrational $\rightarrow$ means $\log_2 3 = \frac{a}{b}$ — $a, b$ are ints, $b \neq 0$, no common factors

can not be done

Know: $\log_2 3 = x$

means: $3 = 2^x$

Pf by contradiction: assume $x$ was rational $\rightarrow$ $x = \frac{a}{b}$ with $\boxed{''}$

$\rightarrow 3 = 2^{a/b} \rightarrow 3^b = 2^a$

So we have a number $n = 3^b = 2^a$ which has 2 prime factorizations. (always false)

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4.3 #43

| | |
|---|---|
| $\gcd(144, 89)$ | so $\overset{a}{144} = (\overset{q_1}{1})\overset{b}{89} + \overset{r_1}{55}$ $- r_2$ |
| $\gcd(89, 55)$ | $89 = (1)55 + 34$ |
| $\gcd(55, 34)$ | $55 = (1)34 + 21$ |
| $\gcd(34, 21)$ | $34 = (1)21 + 13$ |
| $\gcd(21, 13)$ | $21 = (1)13 + 8$ |
| $\gcd(13, 8)$ | $13 = (1)8 + 5$ |
| $\gcd(8, 5)$ | $8 = (1)5 + 3$ |
| $\gcd(5, 3)$ | $5 = (1)3 + 2$ |
| $\gcd(3, 2)$ | $3 = (1)2 + \boxed{1}$ |
| $\gcd(2, 1)$ | $2 = (2)1 + 0$ |

$s_n = s_{n-2} - q_{n-1} s_{n-1}$

$t_n = t_{n-2} - q_{n-1} t_{n-1}$

# Extended Euclidean Algor.

| | q's | s | t | n |
|---|---|---|---|---|
| | | 1 | 0 | $n=0$ |
| $r_0 = q_1 r_1 + r_2$ | 1 | 0 | 1 | $n=1$ |
| $r_1 = q_2 r_2 + r_3$ | 1 | 1 | $-1$ | $n=2$ |
| $r_2 = q_3 r_3 + r_4$ | 1 | $-1$ | 2 | $n=3$ |
| | 1 | 2 | $-3$ | $n=4$ |
| $\vdots$ | 1 | $\vdots$ | $\vdots$ | |

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**Ch 5** prove $\forall n\, P(n)$

until now we had a finite domain for n.

$\rightarrow \quad P(n=1) \wedge P(n=2) \wedge \dots \wedge P(n=k) \equiv \forall n\, P(n)$

prove the finite number of k-cases.

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<u>Now:</u> $1 + 2 + \dots + n = \dfrac{n(n+1)}{2}$

<u>Says:</u> for all n from $n \in \{1, 2, 3, 4, \dots\}$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

This <u>is</u> a $\forall n\, P(n)$ problem **but**

we have a infinite number of cases!

if your cases are $\boxed{\text{well ordered}}$ wait till Discrete 2

we have $\boxed{\text{two}}$ useful tautologies

$$\text{①}\ \left[\underbrace{\left[P(1^{st}\text{ case}) \wedge \forall k\left(P(k^{th}) \to P(k+1^{st})\right)\right]}_{①} \to \underbrace{\forall n P(n)}_{2}\right] \equiv T$$

| ① | ② | ① → ② |
|---|---|---|
| T | T | T |
| ~~T~~ | ~~F~~ | ~~F~~ |
| F | T | T |
| F | F | T |

$\boxed{\text{so}}$ if I can show

$\boxed{\text{Basis step}}$ a) $P(1^{st}\text{ case})$ is true

weak $\boxed{\text{Inductive step}}$ b) $\forall k\left(P(k^{th}) \to P(k+1^{st})\right)$ is true

I will have show $\forall n P(n)$

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Another Tautology

$$\left[P(1^{st}\text{ case}) \wedge \forall k\left[\left(P(1^{st}) \wedge P(2^{nd}) \wedge \ldots \wedge P(k^{th})\right) \to P(k+1^{st})\right]\right] \to \forall n P(n)$$

a) $\boxed{\text{Basis}}$ $P(1^{st}\text{ case})$

b) $\boxed{\text{Strong Inductive Step}}$

$$\left(P(1^{st}) \wedge P(2^{nd}) \wedge \ldots \wedge P(k^{th})\right) \to P(k+1^{st})$$

Prove: $P(n)$: "$1+2+\dots+n = \frac{n(n+1)}{2}$" $n=1,2,\dots$

pf (a) Basis: $P(1^{st}\text{ case}) = P(n=1)$

$P(n=1)$: "$1 = \frac{1(1+1)}{2}$" is true

(b) Inductive Step: Show $P(k^{th}) \boxed{\rightarrow} P(k+1^{st})$

$n=k$ $\quad$ $n=k+1$

assume $P(n=k)$: "$1+2+\dots+k = \frac{k(k+1)}{2}$"

show $P(n=k+1)$: "$1+2+\dots+(k+1) = \boxed{\frac{(k+1)(k+2)}{2}}$" ☆

so assume $1+2+\dots+k = \frac{k(k+1)}{2}$

$1+2+\dots+k+(k+1) = \boxed{\frac{k(k+1)}{2} + (k+1)}$

$\downarrow$ algebra

$1+2+\dots+(k+1) = \frac{(k+1)(k+2)}{2}$