Math 321

Q's / $f(p) = (11p + 4) \bmod 7$

$f^{-1}(c) = (\overline{11}(c-4)) \bmod 7$

$\overline{11}$ = 11's inv under mod 7

gcd(11, 7)

$11 = (1)7 + 4$
$7 = (1)4 + 3$
$4 = (1)3 + \boxed{1}$

$1 = 4 - (3)$ ← $(7-4)$
$1 = 2(4) - 7$
$1 = 2(11-7) - 7$
$1 = (2)11 + (-3)7$

(2): 11's inv. mod 7; (−3): 7's inv mod 11

$$f^{-1}(c) = \boxed{(2(c-4)) \bmod 7}$$

$$f^{-1}(c) = (2c - 8) \bmod 7 = (2c+6) \bmod 7$$

Ch6 Counting + Advanced Counting

6.1 Basics → two important words : And , Or

Problem How many ways to do a task?

ex How many ways to pickup a block of wood (there are 10 blocks) and then paint it (there are 6 colors)?

ex3 How many ways to pick a toy from a set of 6 army men or 21 robots?

AND $|A_1 \times A_2| = |\text{all ordered pairs}| = |A_1| \cdot |A_2|$

ex above $|\text{pick block and pick color}| = 10 \cdot 6 = \boxed{60 \text{ ways}}$

Many sets: $|A_1 \times A_2 \times A_3 \times \cdots \times A_n| = |A_1||A_2| \cdots |A_n|$

Product Rule (uses the word 'and')

OR take from A or B, but $A \cap B = \emptyset$

$|A \cup B| = |A| + |B|$

many sets $|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$

as long as each are disjoint

SUM Rule

ex How many ways to fill out a bubble sheet exam if you allow for ans Ⓐ, Ⓑ, Ⓒ, Ⓓ and no ans. is OK? (we have 10 probs)

8 mult. choice

2 T/F

How to fill it out?

(do prob 1) and (prob 2) and (prob 3) and -- and (prob 10)

5 = fill in bubble

(A) or (B) or (C) or (D) or None

$1 + 1 + 1 + 1 + 1$

2

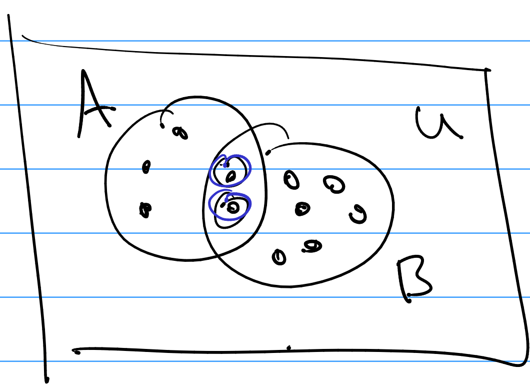
T or F

$1 + 1$

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 2 \cdot 2 = \boxed{5^8 \cdot 2^2}$$

Over Counting

Sum Rule Overcount → Subtraction Rule = Inclusion Exclusion Principle



$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

$$= 5 + 8 - 2$$

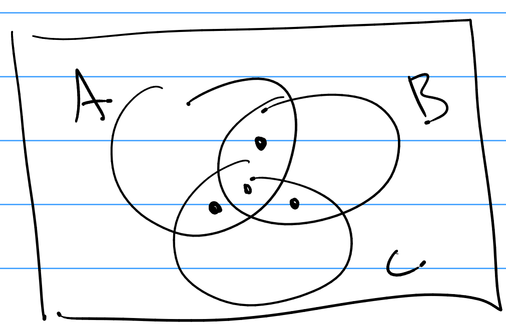
$$= 13 - 2 = \boxed{11}$$

3 sets?

$$|A \cup B \cup C| = |A| + |B| + |C|$$

$$- |A \cap B| - |A \cap C| - |B \cap C|$$

$$+ |A \cap B \cap C|$$



$|A_1 \cup A_2 \cup \dots \cup A_n|$

$= |A_1| + |A_2| + \dots + |A_n|$

$- |A_1 \cap A_2| - |A_1 \cap A_3| - \dots - |A_{n-1} \cap A_n|$

$+ |A_1 \cap A_2 \cap A_3| + \dots + |A_{n-2} \cap A_{n-1} \cap A_n|$

$-$ 4-tuple

$+$ 5-tuple

$\vdots$

$\pm |A_1 \cap A_2 \cap \dots \cap A_n|$

Overcount on product rule $\to$ $\boxed{\text{division rule}}$

$\boxed{\text{ex}}$ 5 kids. Race and we give out 3 medals.

$\to$ How many ways?

total races $= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

4^{th} - 5^{th} = lost $= \boxed{2 \cdot 1}$

real ans $= \dfrac{120}{2} = \boxed{60}$

overcount by mult.

Division Rule There are $\frac{n}{d}$ ways to do a task if it can be done by a procedure that can be carried out in n-ways, and for every way $\underline{w}$, exactly d of the n-ways correspond to way $\underline{w}$.

ex 120 rules

5 people

$$\overbrace{\begin{array}{cccc} 1^{st}, & 2^{nd}, & 3^{rd}, & \text{lose} \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{array}}^{\text{ways}}$$

$$\frac{120}{1 \cdot 1 \cdot 1 \cdot 2} = \boxed{60}$$