Math 321

OR Sum Rule $|A \cup B| = |A| + |B|$ if $|A \cap B| = \emptyset$

Subtraction Rule
(Inclusion / Exclusion Principle)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

AND Product Rule $|A \times B| = |A||B|$

Division Rule

→ task has n-ways to do it. But for a specific way has d-ways for it to happen.

→ $|\text{task}| = n/d$

Ex 6 blocks. 4 red and 2 are white

total ways to arrange 6 blocks?

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$$

Notation b/c products like $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ happen a lot in these problems we will introduce factorial notation...

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$$

Note $\frac{6!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}} = 6 \cdot 5$

for our example but of the 6 blocks = 4 red and 2 white

4 red have $4 \cdot 3 \cdot 2 \cdot 1 = 4!$ arrangements

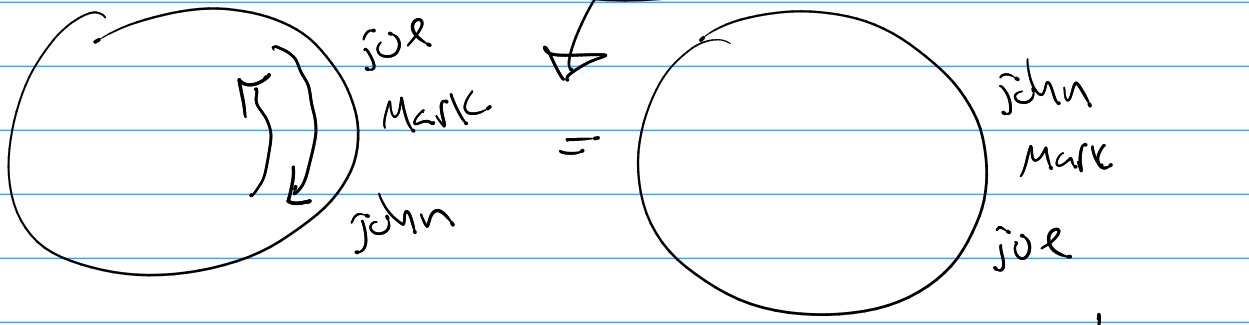
2 white have $2 \cdot 1 = 2!$ arrangements

$$\left| \begin{array}{c} \text{arrangements} \\ \text{of 4 red and 2 white} \end{array} \right| = \frac{6! \; {}^{3}6 \cdot 5}{4! \; 2!} = \boxed{15}$$

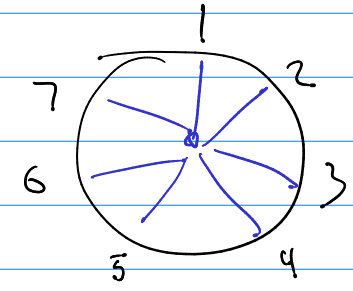
Ex 7 people. How many ways to sit at a straight table?

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7!$$

Ex 7 people. How many ways to sit at a round table if having same people to your left or right is same position.

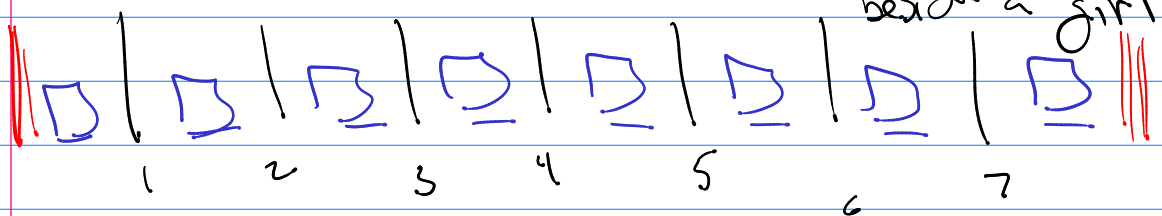


$$\left| \text{total} \right| = \frac{7!}{7 \cdot 2} = 6 \cdot 5 \cdot 4 \cdot 3 = \boxed{360}$$



Ex 7 guys, 5 girls

→ arrange in a line but no girl can stand beside a girl.



$7_0 \cdot 5_0 \cdot \frac{8_0}{5_0 3_0} = 7_0 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \boxed{?}$

Ex integers between and including 10 and 99 = total of 90

How many are divisible by $3_0^?$ (normally a 1/3 of total)

$\frac{90}{3} = \boxed{30}$

How many are div. by $4_0^?$ (normally 1/4 th of total)

$\frac{90}{4} = 22.5 \rightarrow \begin{matrix} 22 \\ 23 \end{matrix} \; ?_0$

Idea: 1, 2, 3, 4 → 4 numbers

div by 3 $\frac{4}{3} = 1.\overline{3} \rightarrow \begin{matrix} \boxed{1} \\ 2 \end{matrix}$ b/c only 3

3, 4, 5, 6 → 4 numbers

div by 3 $\frac{4}{3} = 1.\overline{3} \rightarrow \begin{matrix} 1 \\ \boxed{2} \end{matrix}$ b/c 3, 6

4

10, 11, 12, 13, 14, 15, ..., 96, 97, 98, 99

div by 4 → $\frac{90}{4} = 22.5$ → 22 → 23

$12 + n \cdot 4 = 96$

$n \cdot 4 = 84$

$n = 21$

How many are div. by 2 ?

$\frac{90}{2} = 45$

How many are div. by 2 or 3 ?

|div by 2 or 3| = |by 2| + |by 3| − |by 2 and 3|

(by 2 and 3 → by 6)

|div by 6| = $\frac{90}{6} = \frac{45}{3} = 15$

|div by 2 or 3| = 45 + 30 − 15 = 60