Math 321

6.2 Pigeonhole Principle

idea: More stuff than places to put your stuff then at least one place will have 2 or more things.

Th^m Pigeonhole Principle
given $K \in \mathbb{Z}^+$ (places to put stuff)
and you have $K+1$, or more, objects are placed into the K boxes
then at least one box has at least 2 objects

Th^m Generalized Pigeonhole Principle
if N objects are placed into K boxes
then at least one box has at least $\left\lceil \frac{N}{K} \right\rceil$ objects.

ex

how many socks do you need to pick blindly if you need $\boxed{\text{one pair}}$ of the same color? and colors are red, blue, black, white, yellow

box : color (5)   2 socks of same color
objects : socks

by P.H.P. # socks > # of colors $\rightarrow$ at least one color has at least 2 socks

$\boxed{6}$

ex) not P.H.P. problem that looks like one..

3 - red, 5 - blue, 7 - black, 5 - white, 11 - yellow

→ how many socks to pick blindly to get a red pair?

Worst case (pick 28 of the non-red) + (2-red)

$= \boxed{30}$

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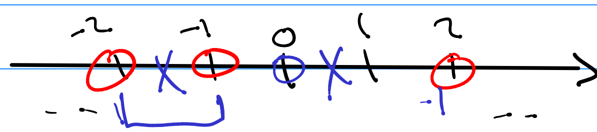
ex) In $\mathbb{R}^1$



we are going to pick only integers. How many integers to pick so at least one of the midpoints of the selected integers is an integer?

① midpt of x_1, x_2 is $\dfrac{x_1+x_2}{2}$

② midpt that is an integer $\dfrac{x_1+x_2}{2} = \text{int}$

→ $x_1 + x_2 = 2(\text{int}) \Leftrightarrow \text{even}$

③ Know
even + even = even
odd + odd = even
even + odd = odd
odd + even = odd

only happens when we have two that have same parity

so · objects : numbers to pick
boxes : parities even or odd (2 of them)

→ pick at least 3 then at least one parity has at least 2 numbers.

→ those two of same parity have int. mpt.

(ex) how many students to have at least 5 with the same 1st and last initials.

objects : students
boxes : initials → 26^2

$$\left\lceil \frac{N}{26^2} \right\rceil = 5 \qquad N = \boxed{4 \cdot 26^2 + 1}$$

6.3 Applied Division Rule

→ $P(n,r) = \dfrac{n!}{(n-r)!}$ permutation

→ $C(n,r) = \dfrac{n!}{r!(n-r)!}$ combination or $\dbinom{n}{r}$

(ex) 6 people = 4 guys + 2 girls

arrangements $= \dfrac{6!}{4!\,2!}$

(ex) 6 objects and you choose 4.

6 objects = 4 chosen + 2 not chosen

$$C(6,4) = \frac{6!}{4!\,2!}$$

(ex) 6 people → 1st place, 2nd place, 3rd place, 4th place

2 losers

$$|\text{arrangements}| = \frac{6!}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 2!} = \frac{6!}{2!}$$

6 people pick 4 winners (with order)

$$P(6,4) = \frac{6!}{2!}$$