

Math 321

Q24 $f_1 + f_3 + f_5 + \dots + f_{2n-1} = f_{2n}$

to $\{0, 1, 1, 2, 3, 5, 8\}$
 $f_n = f_{n-1} + f_{n-2}$

Basis: P(1st case): " $f_1 = f_2$ "
 $1 = 1$ true

Inductive: assume $f_1 + f_3 + \dots + f_{2k-1} = f_{2k}$ I.H.

Show: $f_1 + f_3 + \dots + f_{2k-1} + f_{2k+1} = f_{2k+2}$

So $\boxed{f_1 + f_3 + \dots + f_{2k-1} + f_{2k+1}} \stackrel{\text{I.H.}}{=} \boxed{f_{2k}} + f_{2k+1}$
 $\stackrel{\text{I.H.}}{=} \boxed{f_{2k+2}} \checkmark$
Def of fib. numbers

6.3 Combinations and Permutations

you have n -objects $\rightarrow n!$ arrangements of them.

a) pick r of them
to give place
awards.

1st place
2nd place \dots r th place

b) $(n-r)$ have
no award.

\rightarrow these have $(n-r)!$
arrangements that are
over counts.

$$\text{ans} = \frac{n!}{(n-r)!}$$

Picking r from n with order $\rightarrow \boxed{\frac{n!}{(n-r)!}}$

Def r -permutation of n objects

$$P(n, r) = \frac{n!}{(n-r)!}$$

(pick r with order from n)

$$P(n, r) = \frac{n!}{(n-r)!} = \frac{n(n-1)(n-2) \cdots (n-r+1) \cancel{(n-r)!}}{\cancel{(n-r)!}}$$

$$P(n, r) = \boxed{\begin{array}{c} n \\ | \\ 1 \\ | \\ 1 \end{array}} \cdot \boxed{\begin{array}{c} (n-1) \\ | \\ 1 \\ | \\ 2 \end{array}} \cdots \boxed{\begin{array}{c} (n-r+1) \\ | \\ 1 \\ | \\ r \end{array}} \text{ - factors}$$

Ex 5 people, Pick 3 with order
(3-permutation of 5 objects)

$$P(5, 3) = \frac{5!}{2!}$$

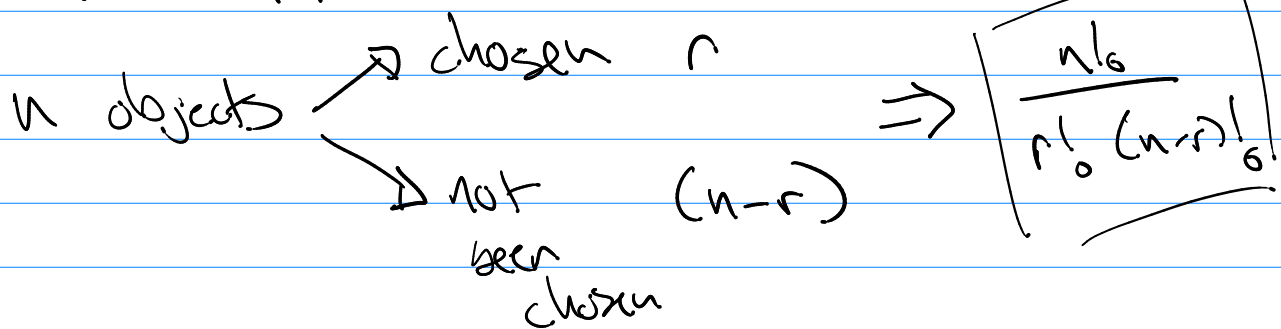
$$P(5, 3) = 5 \cdot 4 \cdot 3$$

Ex Pick 5 player to play basketball from 16.

$$P(16, 5) = \frac{16!}{11!} = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12$$

Combination:

of n objects choose r of them
w/o order.



Def $C(n, r) = \binom{n}{r} = \frac{n!}{r! (n-r)!}$

r -combination of n -objects
of n choose r w/o order

Ex

Softball has 10 players

\rightarrow from 16 people \rightarrow 11 guys + 5 girls

a) Choose 10 players $\rightarrow \binom{16}{10} = \frac{16!}{10! 6!}$
assume no order

b) assign the 10 players $\rightarrow P(16, 10) = \frac{16!}{6!}$
exactly

c) choose 10 players but ^{exactly} 3 must be girls.

choose 3 girls and choose 7 guys

$$C(5, 3) \cdot C(11, 7) = \frac{5!}{3! 2!} \cdot \frac{11!}{7! 4!}$$

a) choose 10 players and at least 3 must be girls.

40 3 girls and 7 guys or 4 girls and 6 guys or 5 girls and 5 guys

$$\binom{5}{3} \cdot \binom{11}{7} + \binom{5}{4} \cdot \binom{11}{6} + \binom{5}{5} \cdot \binom{11}{5}$$

$$\frac{5!}{3!2!} \cdot \frac{11!}{7!4!} + \frac{5!}{4!1!} \cdot \frac{11!}{6!5!} + \frac{5!}{5!0!} \cdot \frac{11!}{5!6!}$$

Def: $0! = 1$

b/c $\binom{5}{5} = \frac{5!}{5!0!} = \frac{1}{0!} \stackrel{\text{or def}}{=} 1$

Note: Combinatorial Proofs.

ex Proof $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

1 way Algebra ?

Algebra

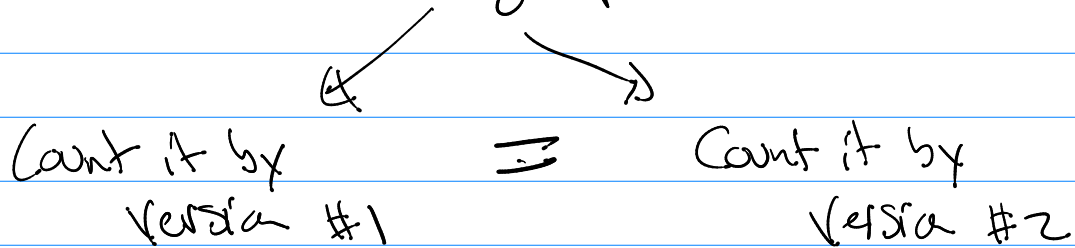
$$\frac{(n+1)!}{r!(n+1-r)!} \stackrel{?}{=} \frac{n!}{r!(n-r)!} + \frac{r \cdot n!}{r(r-1)!(n-(r-1))!}$$

Do Alg.

2nd way

Combinatorial Proof of Country Proof

Describe a counting problem



Ex 3. $\boxed{\text{of } n \text{ pick } r}$

prod rule

overcount division rule

$$\underbrace{n}_{1^{\text{st}}} \cdot \underbrace{(n-1)}_{2^{\text{nd}}} \cdot \dots \cdot \underbrace{(n-r+1)}_{r^{\text{th}}} = \frac{n!}{(n-r)!}$$

So $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

we have $(n+1)$ people. they are n -student plus $\boxed{\text{Mark}}$.

How many ways to choose a group of r -people?

$\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

