Math 321

Q:/ $f_1 + f_3 + f_5 + \dots + f_{2n-1} = f_{2n}$

f_0: 0, 1, 1, 2, 3, 5, 8

$f_n = f_{n-1} + f_{n-2}$

Basis: P(1st case): "$f_1 = f_2$"

$1 = 1$ $\boxed{\text{true}}$

Induction: assume $f_1 + f_3 + \dots + f_{2k-1} = f_{2k}$ I.H.

Show: $f_1 + f_3 + \dots + f_{2k-1} + f_{2k+1} = f_{2k+2}$

So $\boxed{f_1 + f_3 + \dots + f_{2k-1}} + f_{2k+1} \overset{\text{I.H.}}{=} \boxed{f_{2k}} + f_{2k+1}$

$= f_{2k+2}$ ✓

Def of fib. numbers

6.3 Combinations and Permutations

you have n-objects → $n!$ arrangements of them.

a) pick r of them to give place awards. 1st place, 2nd place .. rth place

b) $(n-r)$ have no award. → these have $(n-r)!$ arrangements that are over counts.

$$\text{ans} = \frac{n!}{(n-r)!}$$

Picking r from n with order → $\boxed{\dfrac{n!}{(n-r)!}}$

Def r-permutation of n objects

$$P(n,r) = \frac{n!}{(n-r)!}$$

(pick r with order from n)

$$P(n,r) = \frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-r+1)\cancel{(n-r)!}}{\cancel{(n-r)!}}$$

$$P(n,r) = \underbrace{n(n-1)\cdots(n-r+1)}_{r\text{ - factors}}$$

1, 1, 1
1, 2, r

ex 5 people, Pick 3 with order

(3-permutation of 5 objects)

$$P(5,3) = \frac{5!}{2!}$$

$$P(5,3) = 5 \cdot 4 \cdot 3$$

ex Pick 5 player to play basketball from 16.

$$P(16,5) = \frac{16!}{11!} = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12$$

Combination:

of n objects choose r of them w/o order.

n objects $\to$ chosen r

n objects $\to$ not been chosen $(n-r)$

$$\Rightarrow \boxed{\frac{n!}{r!\,(n-r)!}}$$

Def

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

r-combination of n-objects
of n choose r w/o order

ex

softball has 10 players
$\to$ from 16 people $\to$ 11 guys + 5 girls

a) choose 10 players $\to$ $\binom{16}{10} = \dfrac{16!}{10!\,6!}$
assume no order

b) assign the 10 players $\to$ $P(16,10) = \dfrac{16!}{6!}$

c) choose 10 players but exactly 3 must be girls.

choose 3 girls and choose 7 guys

$$C(5,3) \cdot C(11,7) = \frac{5!}{3!\,2!} \cdot \frac{11!}{7!\,4!}$$

a) choose 10 players and at least 3 must be girls.

So 3 girls and 7 guys or 4 girls and 6 guys or 5 girls and 5 guys

$$\binom{5}{3}\cdot\binom{11}{7} + \binom{5}{4}\cdot\binom{11}{6} + \binom{5}{5}\cdot\binom{11}{5}$$

$$\frac{5!}{3!\,2!}\cdot\frac{11!}{7!\,4!} + \frac{5!}{4!\,1!}\cdot\frac{11!}{6!\,5!} + \frac{5!}{5!\,0!}\cdot\frac{11!}{5!\,6!}$$

?

$$\boxed{\text{Def: } \quad 0! = 1}$$

b/c $\binom{5}{5} = \frac{\cancel{5!}}{\cancel{5!}\,0!} = \boxed{\frac{1}{0!} \overset{\text{by def}}{=} 1}$

Note: Combinatorial Proofs.

ex prove $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$

1 way Algebra?

$$\frac{(n+1)!}{r!\,(n+1-r)!} \overset{?}{=} \frac{n!}{r!\,(n-r)!} + \frac{r\cdot n!}{\underbrace{r(r-1)!}_{r!}\,(n-(r-1))!}$$

After // Do Alg.

$\boxed{2^{nd} \text{ way}}$ Combinatorial Proof $\boxed{\text{or}}$ Counting Proof

describe a counting problem

$$\swarrow \qquad \searrow$$

Count it by version #1 $=$ Count it by version #2

$\boxed{\text{ex}}$. $\boxed{\# \text{ of } n \text{ pick } r}$

prod rule $\swarrow$ $\qquad$ $\searrow$ overcount division rule

$$\underset{1^{st}}{n} \cdot \underset{2^{nd}}{(n-1)} \cdot \ldots \cdot \underset{r^{th}}{(n-r+1)} = \frac{n!}{(n-r)!}$$

$\boxed{\text{So}}$ $$\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$$

we have $(n+1)$ people. they are

n-student plus $\boxed{\text{Mark.}}$

How many ways to choose a group of r-people?

$$\swarrow \qquad \searrow$$

$$\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$$