Math 321

~~Q's~~ 6.2 #7 $n \in \mathbb{Z}^+$

n- consecutive integers : $k, k+1, k+2, \dots, k+n-1$

to be divisible by $n \to a \bmod n = 0$

mod n has $0, 1, 2, \dots, n-1$ (we have n- remainders)

$\to k \bmod n, \; k+1 \bmod n = k \bmod n + 1, \dots, k \bmod n + (n-1)$

we have n uniq. remainders for n possible n-remainders

exactly one is div. by n

6.3 #21 A, B, C, D, E, F, G

a) $\boxed{BCD}$, A, E, F, G permutations of 5 things $\to \boxed{5!}$

b) $\boxed{CFGA}$, B, D, E $\to \boxed{4!}$

ex A, B, C, D, E, F, G $\boxed{\text{allow reuse}}$

① 7 symbol strings $\to 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^7$

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ex n-people → task: make a committee of K and then pick a leader of them.

① $\binom{n}{k} \cdot K = \boxed{K\binom{n}{k}}$

② $\boxed{n \cdot \binom{n-1}{k-1}}$

b/c they counted the same thing → $\boxed{K\binom{n}{k} = n\binom{n-1}{k-1}}$

6.4 $(x+y)^n$

Binomial theorem:

$(x+y)^6 = 1\,x^6y^0 + 6\,x^5 \cdot y^1 + 15\,x^4 \cdot y^2 + 20\,x^3 \cdot y^3 + 15\,x^2 \cdot y^4$

$+ 6\,x^1 y^5 + 1 \cdot y^6$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

→ 1 6 15 20 15 6 1

$(\square + \Delta)^n = \underbrace{(\square+\Delta)\cdot(\square+\Delta)\cdot \ldots \cdot(\square+\Delta)}_{n\text{-factors}}$

take a $\square$ or Δ n-times

if I do that I will have n-objects (□'s and △'s)

or ① $\frac{n!}{n!\,0!}$ □^{n} △^{0} = $\binom{n}{0}$ □^{n} △^{0}

or ② $\frac{n!}{(n-1)!\,1!}$ □$^{n-1}$ △^{1} = $\binom{n}{1}$ □$^{n-1}$ △^{1} +

or ③ □$^{n-2}$ △^{2} = $\binom{n}{2}$ □$^{n-2}$ △^{2} +

⋮

or ⓝ □^{0} △^{n} = $\binom{n}{n}$ □^{0} △^{n} +

Binomial Th^{m}

$$(□ + △)^n = \binom{n}{0} □^n △^0 + \binom{n}{1} □^{n-1} △^1 + \cdots + \binom{n}{j} □^{n-j} △^j + \cdots + \binom{n}{n} □^0 △^n$$

ex $(3x-2y)^4$

$$= \binom{4}{0}(3x)^4(-2y)^0 + \binom{4}{1}(3x)^3(-2y)^1 + \binom{4}{2}(3x)^2(-2y)^2 + \binom{4}{3}(3x)^1(-2y)^3 + \binom{4}{4}(3x)^0(-2y)^4$$

= do the algebra!

ex $(1+1)^n = 2^n$

$$= \binom{n}{0}(1)^n(1)^0 + \binom{n}{1}(1)^{n-1}(1)^1 + \binom{n}{2}(1)^{n-2}(1)^2 + \cdots + \binom{n}{n}(1)^0(1)^n$$

$$= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$$

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} \sim + \binom{n}{n}$$