Math 321

Q's/ $$\binom{2n}{n+1} + \binom{2n}{n} = \frac{1}{2}\binom{2n+2}{n+1}$$

techs to try (1) Algebra

$$\frac{(2n)!}{(n+1)!\,(2n-(n+1))!} + \frac{(2n)!}{(n)!\,(2n-n)!} \overset{?}{=} \frac{1}{2}\,\frac{(2n+2)!}{(n+1)!\,(2n+2-(n+1))!}$$

$$\frac{n\,(2n)!}{\underbrace{(n+1)!}_{(n+1)\,n!}\,\underbrace{\boxed{n(n-1)!}}_{n!}} + \frac{(n+1)\,(2n)!}{\underbrace{(n+1)(n)!}_{(n+1)!}\,(n)!} \overset{?}{=} \frac{(2n+2)!}{2\,(n+1)!\,(n+1)!}$$

$$\frac{n\,(2n)!}{(n+1)!\,n!} + \frac{(n+1)\,(2n)!}{(n+1)!\,n!} \overset{?}{=} \frac{(2n+2)!}{2\,(n+1)!\,(n+1)!}$$

$$\frac{(2n+1)\quad(2n)!}{(n+1)!\quad n!} \overset{?}{=} \quad ''$$

$$\frac{(2n+1)!}{(n+1)!\,n!} \overset{?}{=} \boxed{\frac{(2n+2)!}{2\boxed{(n+1)!}\,(n+1)!}}$$

$$\frac{(2n+1)!}{(n+1)!\,n!} \overset{?}{=} \frac{\cancel{(2n+2)}\cdot(2n+1)!}{\cancel{2(n+1)}\cdot n!\,(n+1)!} \quad ✓$$

tech #2

Counting proof

$\boxed{\text{task}}$!

Count by way #1

Count by way #2

$$\boxed{2\binom{2n}{n+1}+2\binom{2n}{n}} = \boxed{\binom{2n+2}{n+1}}$$

$$\boxed{\binom{n+1}{k} = \binom{n}{k-1}+\binom{n}{k}}$$

8.2

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

Soln $a_n = r^n$ but we will find that there are k r's to get.

part 1 find the r's

plug in r^n $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

$r^n = c_1 r^{n-1} + c_2 r^{n-2} + \dots + c_k r^{n-k}$

by normal algebra

$$r^k = c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_k$$

$$r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k = 0$$

↳ solve the polynomial to find $\underline{r's}$

ex

$$a_n = 2a_{n-2} + 3a_{n-4}$$

$a_n = r^n$

$$r^n = 2r^{n-2} + 3r^{n-4}$$

$$\cancel{r^n} = 2\frac{\cancel{r^n}}{r^2} + 3\frac{\cancel{r^n}}{r^4}$$

$$r^4 = 2r^2 + 3$$

$$\boxed{r^4 - 2r^2 - 3 = 0}$$

↳ solve to get the 4 - $\underline{r's}$

$$a_n = 2a_{n-2} + 3a_{n-4}$$

$$a_n = 0a_{n-1} + 2a_{n-2} + 0a_{n-3} + 3a_{n-4}$$

↳ $$\boxed{r^4 - 0r^3 - 2r^2 - 0r - 3 = 0}$$

$$(r^2)^2 - 2r^2 - 3 = 0$$

$$(r^2 - 3)(r^2 - 1) = 0$$

$$r = \sqrt{3} \quad r = -\sqrt{3} \quad r = i \quad r = -i$$

Part 2

You found the r's

$$\underline{a_n = r_1^n}, \quad \underline{a_n = r_2^n}, \quad \dots, \quad \underline{a_n = r_k^n}$$

→ the de soln is ... poly of n with the multiplicity of r_1 # of terms

$$\begin{aligned} a_n = {} & (\boxed{})\, r_1^n \\ & + (\qquad\qquad)\, r_2^n \\ & \vdots \\ & + (\qquad\qquad)\, r_k^n \end{aligned}$$

$$(x+1)(x+1)(x-2)(x+3) = 0$$

$$\text{roots} \quad \underbrace{x=-1 \quad x=-1}_{\text{mult. of } 2} \quad \underbrace{x=2}_{\substack{\text{multiplicity} \\ \text{of } 1}} \quad \underbrace{x=-3}$$

$$\text{poly of } n = c_0 + c_1 n + c_2 n^2 + \dots + c_k n^k$$

ex $a_n = \boxed{\text{blah}}$

the r's are $r_1 = 2,\ r_2 = 2,\ r_3 = 2,\ r_4 = (-1),$
$r_5 = (-1),\ r_6 = 7$

$$\boxed{\begin{aligned} a_n = &(c_0 + c_1 n + c_2 n^2)(2)^n \\ &+ (c_3 + c_4 n)(-1)^n \\ &+ (c_5)(7)^n \end{aligned}}$$

general soln.

if you are given Basis Values you can then find the constants.

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ex last class

$f_n = f_{n-1} + f_{n-2} \quad f_0 = 0 \quad f_1 = 1$

① $r^2 - r - 1 = 0$
$\to r = \dfrac{1 \pm \sqrt{5}}{2}$

② $f_n = c_1 \left(\dfrac{1+\sqrt{5}}{2}\right)^n + c_2 \left(\dfrac{1-\sqrt{5}}{2}\right)^n$

③ give $f_0 = 0 \quad f_1 = 1$ plug in

$c_1 = \dfrac{1}{\sqrt{5}} \qquad c_2 = -\dfrac{1}{\sqrt{5}}$

Specific Soln $f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$