Math 321

Q's | 8.1 #27

0, 1, 2 no 11?

Remember

0, 1 no 11

length n = a_n # of ways with no 11

$a_n \to 0\,b_2 b_3 \dots b_n$

no 11 in it = a_{n-1}

$a_n \to 10\,b_3 \dots b_n$

no 11 in it = a_{n-2}

$$a_n = a_{n-1} + a_{n-2}$$

Real prob

$t_2\ t_3\ \dots\ t_n$ no rr = a_{n-1}

or

$t_2\ t_3\ \dots\ t_n$ no rr = a_{n-1}

or

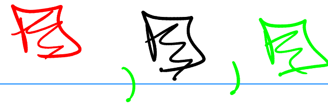
2

$t_3\ t_4\ \dots\ t_n$ no rr = a_{n-2}

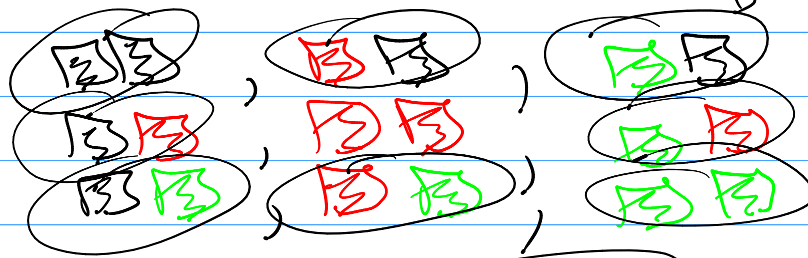
$$a_n = 1\cdot a_{n-1} + 1\cdot a_{n-1} + 2\cdot a_{n-2}$$

$$a_n = 2a_{n-1} + 2a_{n-2}$$

Basis

$a_1 = 3$ 

all of 2-tiles

$a_2 = 8$ 

no rr

$a_n = 2a_{n-1} + 2a_{n-2}$ $\quad \boxed{a_1 = 3, \; a_2 = 8}$

seq: $3, 8, 22, 60, 164, \ldots$

a_1, a_2, a_3, a_4, a_5

$a_n = 2a_{n-1} + 2a_{n-2}$ $\quad \boxed{a_0 = 1, \; a_1 = 3}$

Seq $1, 3, 8, \ldots$

6.2 Solve: $a_n = 2a_{n-1} + 2a_{n-2}$ $\quad a_0 = 1, a_1 = 3$

$a_n = r^n$ (find the r's)

Solve: $r^2 - 2r - 2 = 0$

$$r = \frac{2 \pm \sqrt{4 + 8}}{2} = 1 \pm \sqrt{3}$$

$r_1 = (1 + \sqrt{3})$ $\quad r_2 = (1 - \sqrt{3})$

$$a_n = (c_1)(1 + \sqrt{3})^n + (c_2)(1 - \sqrt{3})^n$$

$$a_n = c_1(1+\sqrt{3})^1 + c_2(1-\sqrt{3})^1$$

$$a_0 = 1 \qquad a_1 = 3$$

$$1 = c_1 + c_2 \rightarrow \boxed{c_2 = 1 - c_1}$$

$$3 = c_1(1+\sqrt{3}) + c_2(1-\sqrt{3})$$

$$\rightarrow 3 = c_1(1+\sqrt{3}) + (1-c_1)(1-\sqrt{3})$$

$$3 = 2\sqrt{3}\,c_1 + 1 - \sqrt{3}$$

$$\boxed{\frac{2+\sqrt{3}}{2\sqrt{3}} = c_1} \qquad \boxed{c_2 = 1 - \frac{2+\sqrt{3}}{2\sqrt{3}}}$$

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ex

$$a_n = (\qquad) \rightarrow \text{poly} \rightarrow r^3$$

(multiplicity example)

$r_1 = -1,\ r_2 = -1$

$r_3 = 2$

$$a_n = (c_1 + c_2 n)(-1)^n$$

$$+ (c_3)(2)^n$$

Exam 4 (11 probs)

6.1 2 probs (sum, product, subtraction, division rules)

① string type (passwords, license plates, etc)

② # of ints that divide ...

ex (how many ints from 50 to 100 (inclusive) or div. by 7)

6.2 2 probs (pigeonhole principle)

① generalized type (remainders, socks, initials..)

② Integer midpoint prob.

6.3 2 probs ($P(n,r)$. , $C(n,r)$)

① pick / choose a team

Note: useful to know

$|\text{all teams}| = |\text{no guys}| + |\text{1 guy}| + |\text{2 guys}| + \ldots$

② choose for strings

$\boxed{6.4}$ 2 probs $(\square + \triangle)^n = \sum_{k=0}^{n} \binom{n}{k} \square^{n-k} \triangle^k$

(1) Use it.

ex $\left(2 - 3/x\right)^{12}$ → all of it?
→ 5^{th} term
→ coeff of x^{-7}

(2) Prove Pascal's Identity

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

$\boxed{8.1}$ 1 prob (Apps of rec. rel.)

(1) Set it up → relation
→ basis

(ex Start of today)

$\boxed{8.2}$ 2 probs (lin. homog. rec. rel. of deg k with const. coeff)

(1) $\boxed{\text{Solve}}$ (full) degree = 2

deg = 4

(2) Solve up to
Find r's put them into →

$a_n = (\quad) r_1^n$
$+ (\quad) r_2^n$
$\ldots + (\quad) r_k^n$