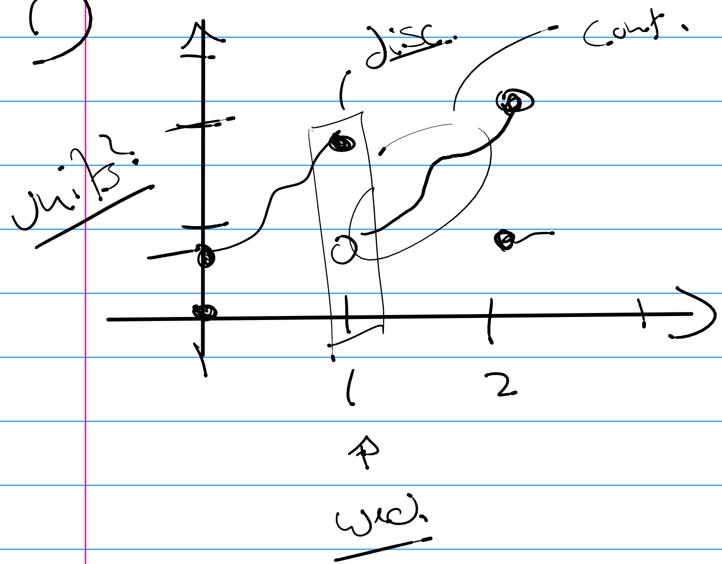
Math 242

$\boxed{\text{Exam 1}}$ out of 140

1)



Domain: 5 wks?

Codomain: unit of height

Range: unit of height for cut height (min) ~ max growth

2) even: $f(-x) = f(x)$, odd: $f(-x) = -f(x)$

a) $f(x) = \dfrac{x}{x^2+1}$ $\quad f(\boxed{-x}) = \dfrac{\boxed{-x}}{\boxed{-x}^2+1} = -\dfrac{x}{x^2+1}$

$= -f(x)$

$\boxed{\text{odd}}$

b) $f(-x) = \dfrac{-x}{-x+1} = \dfrac{x}{1-x}$ $\quad \boxed{\text{neither}}$

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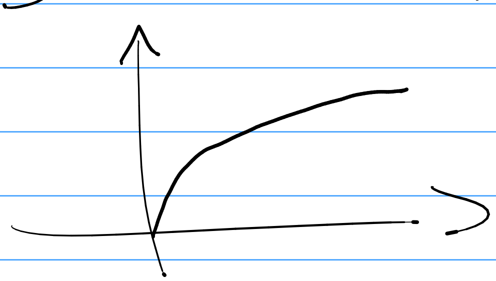
3) a) $s^{1/4}$ (root) domain $[0, \infty)$



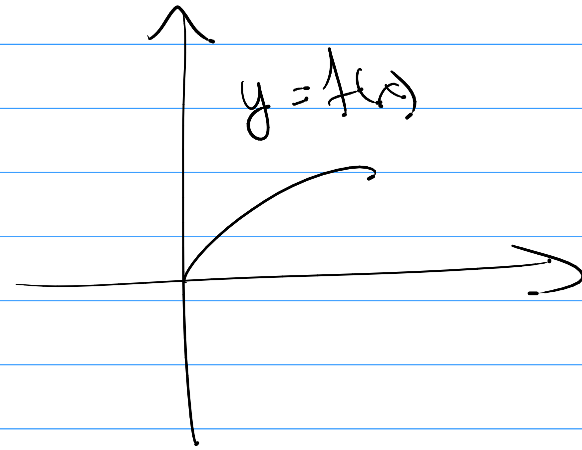
b) $\dfrac{t^3}{t^2-1}$ (rational) $\mathbb{R} - \{\pm 1\}$

3d) $x^3(1-x^2)$ Poly. degree 5

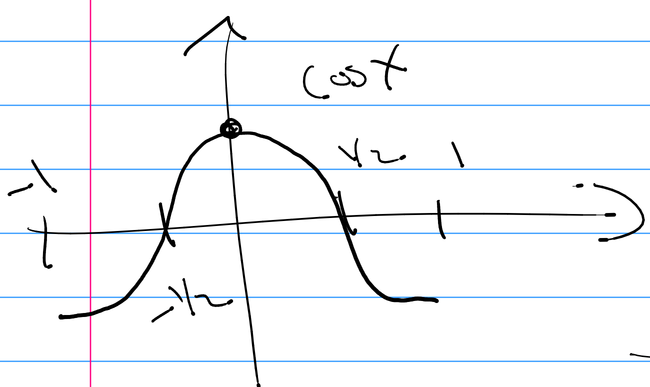
$-x^5+x^3$

4) $x^{1/4} = \sqrt[4]{x}$

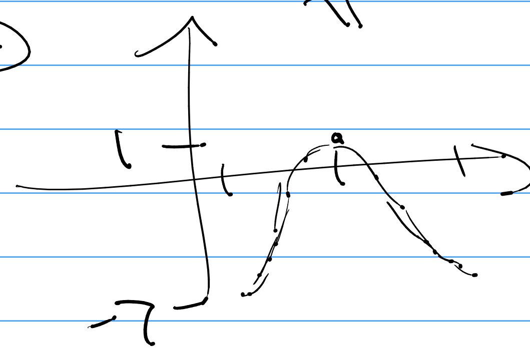
$y = x^{1/4}$



5) $4\cos(\pi(x-2))-3$



$\frac{2\pi}{\pi} = 2$



6) $f(x) - g(x) = \frac{(3x+5)}{1} - \frac{(x+1)}{(x-1)}$

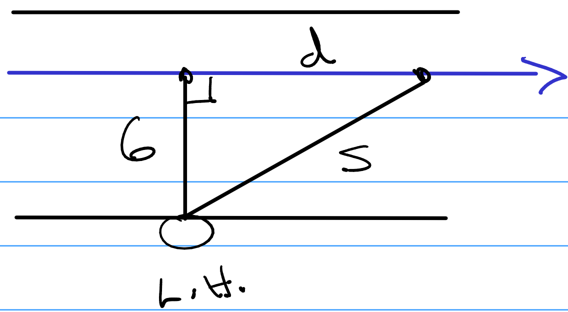
$\boxed{\text{Algebra!}}$

$(g\circ f)(x) = g(f(x)) = g(3x+5)$

$= \frac{(3x+5)+1}{(3x+5)-1} = \text{etc.}$

7)

a)



$$s^2 = 36 + d^2$$

$$s = (36 + d^2)^{1/2}$$

b) 30 km/h $\qquad d = 30t$

etc.

~~8)~~ was supposed to be $\lim_{x \to 5} f(x)$

+10

E10 = 100%

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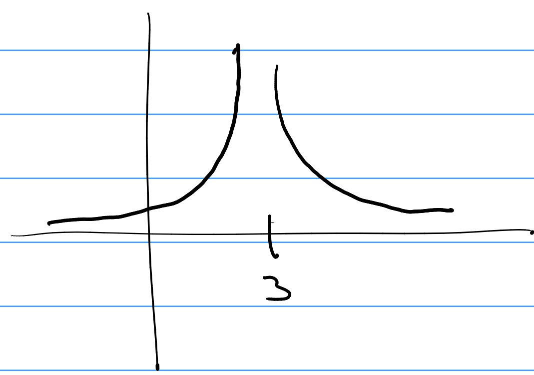
10)



$$\frac{2}{|x-3|}$$

$$\lim_{x \to 3} f(x) = +\infty$$

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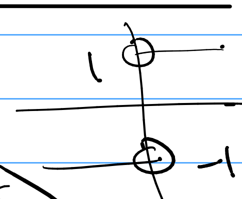
4) $\lim_{x \to a} f(x)$ exists if

$$\lim_{x \to a^+} f(x) = L = \lim_{x \to a^-} f(x)$$



$$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$$

not same!

12) limit laws $\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \lim_{x\to c} g(x)$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ as long as .. $\lim_{x\to c} g(x) = 0$

13) $\lim_{x\to 5} \frac{x^2-6x+5}{x-5} = \lim_{x\to 5} \frac{(x-1)\cancel{(x-5)}}{\cancel{(x-5)}} = \lim_{x\to 5} x-1$

Notation

$= 5-1 = \boxed{4}$

14) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3} \frac{\frac{3-x}{3x}}{\frac{x-3}{1}} = \lim_{x\to 3} \frac{3-x}{3x} \cdot \frac{1}{x-3}$

$= \lim_{x\to 3} \frac{-\cancel{(x-3)}}{3x\cancel{(x-3)}} = \boxed{\frac{-1}{9}}$

16) $\lim_{x\to 1} 3x+5 = 8$

$\forall \varepsilon>0\ \exists \delta>0$ $\boxed{\text{if } 0<|x-1|<\delta, \text{ then } |(3x+5)-8|<\varepsilon}$

Scratch $|3x+5-8|<\varepsilon,\ |3x-3|<\varepsilon,\ |x-1|<\varepsilon/3$

Pf: let $\delta = \varepsilon/3$ etc

1a) $\tan x$

$\sqrt{4-x^2}$

disc.? $\tan x = \dfrac{\sin x}{\cos x}$ disc. @ $\cos x = 0$

$x \dots$

$\dfrac{1}{\sqrt{4-x^2}}$ disc @ $4 - x^2 \le 0$

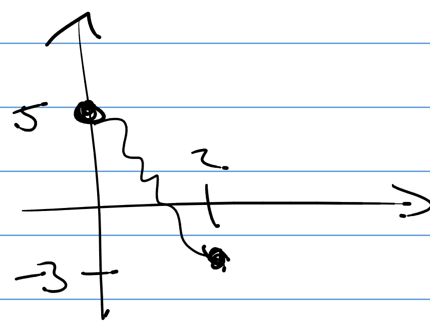
20) $\lim\limits_{x\to\pi} \sin(x + \sin(x)) = \sin(\pi + \sin\pi)$

$= 0$

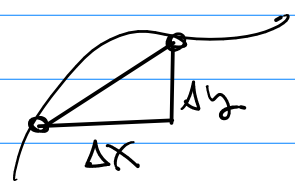
21) $f(x) = \boxed{x^5 - 10x^2 + 5}$

$f(0)$

$f(2)$



Derivatives



rate of change = slope of secant

$= \dfrac{\Delta y}{\Delta x}$

instantaneous rate of change $\lim\limits_{\Delta x\to 0} \dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx}$

Derivative as limit

$$\boxed{\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

Notation:

$f'(x) = y' = \dfrac{dy}{dx} = \dfrac{d}{dx} y = \dfrac{d}{dx}[f(x)] = D[f(x)] = D_x[f(x)]$

b/c $f(x) \rightarrow \boxed{\text{Derivative}} \rightarrow f'(x)$ is a function

① evaluate $f'(x)$ @ $x=a$

Notation: $f'(a) = \left.\frac{dy}{dx}\right|_{x=a}$

ex $f(x) = \frac{x^2-1}{2x-3}$ find $f'(1) = \left.\frac{d}{dx}\left[\frac{x^2-1}{2x-3}\right]\right|_{x=1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{(x+h)^2-1}{2(x+h)-3} - \frac{x^2-1}{2x-3}}{h}$$

$$= \lim_{h\to 0} \frac{\left((x+h)^2-1\right)(2x-3) - (x^2-1)(2(x+h)-3)}{h(2(x+h)-3)(2x-3)}$$

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2-1)(2x-3) - (x^2-1)(2x+2h-3)}{h(2x+2h-3)(2x-3)}$$

$$= \lim_{h\to 0} \frac{[\cancel{2x^3} + 4x^2h + 2xh^2 \cancel{-2x} \cancel{-3x^2} - 6xh - 3h^2 \cancel{+3}] - [\cancel{2x^3} + 2x^2h \cancel{-3x^2} \cancel{-2x} - 2h \cancel{+3}]}{h(2x+2h-3)(2x-3)}$$

$$= \lim_{h\to 0} \frac{2x^2\cancel{h} + 2xh^{\cancel{2}} - 6x\cancel{h} + 2\cancel{h} - 3h^{\cancel{2}}}{\cancel{h}(2x+2h-3)(2x-3)}$$

$$= \frac{2x^2-6x+2}{(2x-3)(2x-3)} = \boxed{\frac{2(x^2-3x+1)}{(2x-3)^2} = f'(x)}$$

so $f'(1) = \dfrac{2(1-3+1)}{(2-3)^2} = \boxed{-2}$

Differentiable Functions

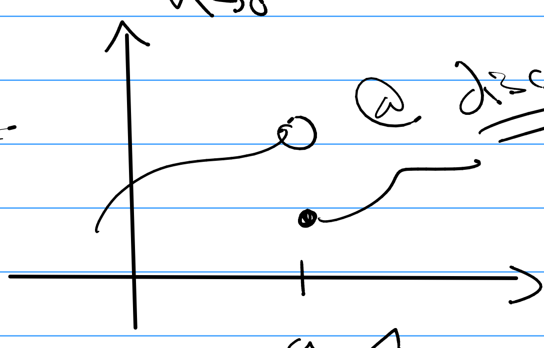
(1) f is differentiable @ $x=a$ if $f'(a)$ exists

(2) f is differentiable on an open interval if $f'(x)$ exists for all x in the open interval

When is f $\boxed{\text{not}}$ differentiable?

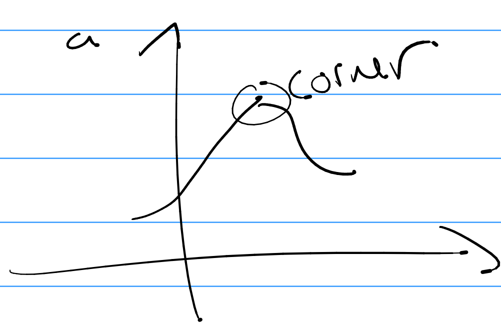
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

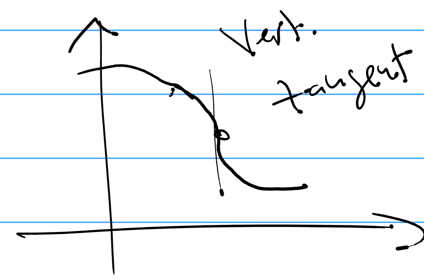
geometrically



@ discontin. → not diff.

f is cont. but $f'(x)$ dne





Thm: If f is differentiable @ $x=a$ then f is continuous @ $x=a$.

→ Smooth functions.

(2) Multiple Derivatives (higher-order derivatives)

$$\frac{d}{dx}[f(x)] = f'(x) \text{ is a function}$$

$$D_x[f'(x)] = \frac{d}{dx}\left[\frac{d}{dx}[y]\right] = \frac{d^2y}{dx^2}$$

$$= D_x^2[f(x)]$$

n-th Derivatives

$$f^{(n)}(x) = y^{(n)} = D_x^{(n)}[f(x)] = \frac{d^n y}{dx^n}$$

Ex: $f(x)$ = position

x-time: $f'(x)$ = velocity

$f''(x)$ = acceleration

$f^{(3)}(x)$ = jerk

$f^{(4)}(x)$ = snap

$f^{(5)}(x)$ = crackle

$f^{(6)}(x)$ = pop

ex of formula

$$\boxed{3x^2 - 4x + 17 = 0}$$

$$\rightarrow ax^2 + bx + c = 0$$

algebra

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$