Math 321

Q's! Homework

① All assigned already (see syllabus)

② Hand in?

M	S	M→S	¬M∨S	(M→S)↔(¬M∨S)
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Def: two compound propositions, C and d, are logically equivalent if $C \leftrightarrow d$ is a tautology.

Notation: $C \equiv d$ or use $C \Leftrightarrow d$

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above table shows $(p \to q) \equiv (\neg p \vee q)$

$(\square \to \Delta) \equiv (\neg \square \vee \Delta)$

Ex $(S \leftrightarrow r) \equiv (S \wedge r) \vee (\neg S \wedge \neg r)$

show $(S \leftrightarrow r) \leftrightarrow [(S \wedge r) \vee (\neg S \wedge \neg r)]$

is a tautology

S	r	S ⊕ r	S ∧ r	(¬S ∧ ¬r)	(S ∧ r) ∨ (¬S ∧ ¬r)	LS ⇔ RS
T	T	F	T	F	T	T
T	F	T	F	F	F	T
F	T	T	F	F	F	T
F	F	F	F	T	T	T

(S)

(S)

$$(S \oplus r) \equiv [(S \wedge r) \vee (\neg S \wedge \neg r)]$$

allows us to show $\square \equiv \Delta$ by

(1) show $\square, \Delta$ are both true @ exact same time.

or

(2) show $\square, \Delta$ are both false @ exact same time.

(ex) $(p \vee q) \equiv (q \vee p)$

table

p	q	p∨q	q∨p	(p∨q)⇔(q∨p)
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(a) Argue a) when is $p \vee q$ false? both are false.

b) when is $q \vee p$ false? both are false

same conditions

so $(p \vee q) \equiv (q \vee p)$

useful logical equiv. (laws)

(1) $p \vee \neg p \equiv T$ negation laws
$p \wedge \neg p \equiv F$

(2) Identity laws $p \wedge T \equiv p$
$p \vee F \equiv p$

(3) $p \vee T \equiv T$ Domination laws
$p \wedge F \equiv F$

(4) Idempotent laws $p \vee p \equiv p$
$p \wedge p \equiv p$

(5) $\neg(\neg p) \equiv p$ double negation

(6) $p \vee q \equiv q \vee p$ commutative laws
$p \wedge q \equiv q \wedge p$

(7) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ assoc.
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(8) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$
$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$

$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

(9) $(p \rightarrow r) \equiv \neg p \vee r$

(10) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ DeMorgan's laws

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

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other implication equiv

$$(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$$

Name: $p \rightarrow q$

its converse is $q \rightarrow p$

its inverse is $\neg p \rightarrow \neg q$

its contrapositive is $\neg q \rightarrow \neg p$

to do?

1. Show
2. use