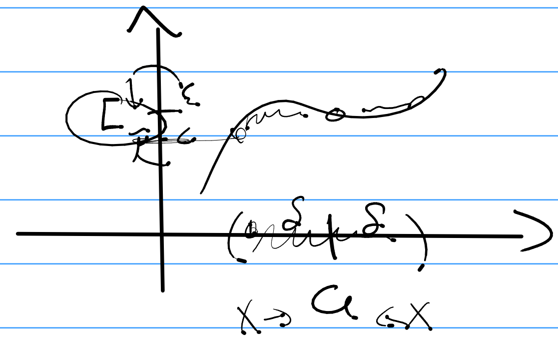
Math 321

Quantification

Propositional function

$$\lim_{x\to a} f(x) = L$$



$\forall \varepsilon > 0 \quad \exists \delta > 0$

if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$

P

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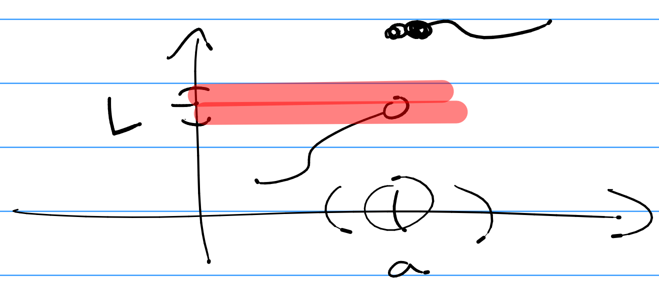
when do we say $\lim_{x\to a} f(x)$ d.n.e. ?

(this is not true)

$$\neg\left(\forall \varepsilon > 0 \ \exists \delta > 0 \quad 0 < |x-a| < \delta \rightarrow |f(x)-L| < \varepsilon\right)$$

$$\equiv \exists \varepsilon > 0 \ \forall \delta > 0 \quad \neg\left(\neg(0 < |x-a| < \delta) \vee |f(x)-L| < \varepsilon\right)$$

$$\equiv \exists \varepsilon > 0 \ \forall \delta > 0 \quad \left(0 < |x-a| < \delta \wedge \neg(|f(x)-L| < \varepsilon)\right)$$



Argument

seq of statements end with a conclusion.

premise

$(\text{statement} \wedge \text{statement} \wedge \ldots \wedge \text{statement}) \rightarrow \text{conclusion}$

Valid if it is always true

$\square$	$\triangle$	$\square \rightarrow \triangle$
T	T	T
T	F	F
F	T	T
F	F	T

If you have true statements force conclusion to be true → tautology!

(Valid)

Notation:

premise
premise
⋮
premise

$\therefore$ conclusion

Argument form.

premises, conclusion have prop. variables

ex $[(p \rightarrow q) \wedge p] \rightarrow q$

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

If a tautology, it is $\boxed{\text{Valid}}$

$$[(p \to q) \wedge p] \to q \equiv \neg[(p \to q) \wedge p] \vee q$$

$$\equiv [\neg(p \to q) \vee \neg p] \vee q$$

$$\equiv \neg(p \to q) \vee (\neg p \vee q) \equiv \neg(p \to q) \vee (p \to q)$$

$$\equiv T$$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

affirming the hypothesis / modus ponens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

denying the conclusion / modus tollens

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

hypothetical syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

disjunctive syllogism

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

addition

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

simplification

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not a tautology

Note! use an Invalid argument form as if it was a rule of inference

→ It is called a **fallacy**

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

fallacy of affirming the conclusion

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

fallacy of denying the hypothesis

$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \hline \therefore q \lor r \end{array}$$

resolution

Quantification: actual measures ⇄ general statements of all, some

$$\begin{array}{l} \forall x\, P(x) \\ \hline \therefore P(c) \text{ any } c \end{array}$$

univ. instantiation

$$\begin{array}{l} P(c) \text{ for all } c \\ \hline \therefore \forall x\, P(x) \end{array}$$

univ. generalization

$\exists x\, P(x)$

$\therefore P(c)$ for some c

Existential Instantiation

$P(c)$ for some c

$\therefore \exists x\, P(x)$

Existential Generalization