

# Math 321

has predicate 1, predicate 2  $\forall x (\text{raven}(x) \rightarrow \text{black}(x))$   
 $\uparrow$   $\uparrow$   
raven black

all ravens are black  $\forall x (\text{raven}(x) \rightarrow \text{black}(x))$

U.D. is all birds  $b_1, b_2, b_3, \dots, b_n$

True?  $(\text{raven}(b_1) \rightarrow \text{black}(b_1)) \wedge \dots$

False?  $\neg \forall x (\text{raven}(x) \rightarrow \text{black}(x))$  is true

$\exists x (\text{raven}(x) \wedge \neg \text{black}(x))$

Counterexample.

1.6 #35  
"superhero"  
 $p_1$  Able  $\wedge$  willing  $\rightarrow$  stop evil  
 $p_2$   $\neg$  Able  $\rightarrow$  weak  
 $p_3$   $\neg$  willing  $\rightarrow$  bad  
 $p_4$   $\neg$  stop evil  
 $p_5$  exist  $\rightarrow$  ( $\neg$ weak  $\wedge$   $\neg$ bad)  
C  $\therefore \neg$  exist

Valid  
Argument Form

Goal  $\rightarrow$  speak true things

use true premises with tautologies to get to true conclusions.  
(logical equiv)  
(valid argument forms)

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## Axiomatic Method

Terms      Undefined terms  $\rightarrow$  Define terms

Propositions

axioms or postulates  $\rightarrow$  "show" statements are true  
accept  $\&$  that they are true  $\uparrow$  proof

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## Proving

Statement starts as a "conjecture"

$\Delta$  maybe true

conjecture  $\xrightarrow{\text{disprove}}$  not always true

prove  $\rightarrow$  show it is always true.

New Name:

fact, result

theorem

lemma  $\rightarrow$  show?

obvious result because of  
a theorem, called a  
corollary

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# Prove stuff

typical conjecture is ...

type #1

$$\square \rightarrow \triangle$$

(ex) if  $n$  is even  
then  $n^2$  is even.

techniques to prove  $\square \rightarrow \triangle$  type conjectures

#1

assume  $\square$  is true [show]  $\triangle$  is true.

(direct proof)

use rules of

logical equiv.  
truths of math

(ex) "n is even  $\rightarrow$   $n^2$  is even"

[DP]

assume  $n$  is even. So  $n = 2 \cdot k$ ,  $k$  is any integer.

$$\begin{aligned} \text{b/c } n = 2k \text{ then } n^2 &= (2k)^2 \rightarrow n^2 = 4k^2 \\ \rightarrow n^2 &= 2(\underbrace{2k^2}_{\substack{\text{integer}}}) \text{ so } \underline{\underline{n^2 \text{ is even}}} \end{aligned}$$

