Math 321

Q's/ (1.5 #3) $Q(\square, \triangle)$: "$\square$ emailed $\triangle$"

u.D. for both x,y are students in class

b) $\exists x \forall y \, Q(x,y)$

d) $\exists y \forall x \, Q(x,y)$

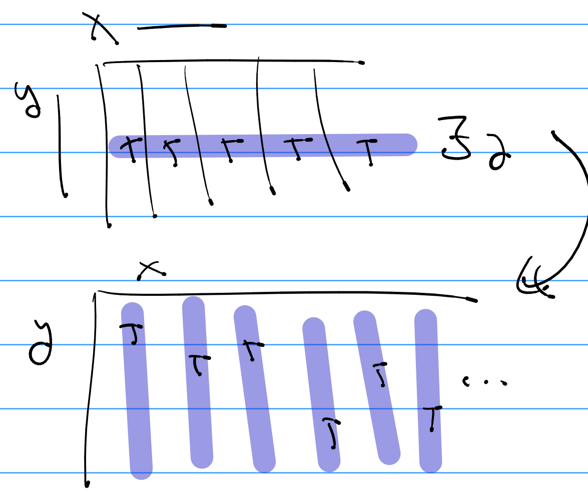
$\exists s \forall t \, \text{Email}(s,t)$

$\forall x \exists y \, Q(x,y)$

$Q(x,y)$: "$x+y=0$"

$\rightarrow$ $\exists y \forall x \, Q(x,y)$ $\rightarrow$

$\rightarrow$ $\forall x \exists y \, Q(x,y)$



Proofs Continued

Conjecture type #1 — show $\square \rightarrow \triangle$ is true.

Note: enthymeme: non-stated premise

proof tech #1 — direct proof

assume $\square$ is true, show $\triangle$ is true (ex) n is odd $\rightarrow n^2$ is odd   (n, n^2 are integers)

pf assume n is odd, $n = 2(k) + 1$, k is an integer

$$n^2 = (2k+1)^2$$
$$n^2 = 4k^2 + 4k + 1$$
$$n^2 = 2(2k^2 + 2k) + 1$$ is $2 \cdot \text{int} + 1$ so it is odd

∎

---

tech #2 show $\square$ is simply false (**ignore** $\triangle$)

Vacuous So $\square \rightarrow \triangle$ is true

prove if $\boxed{1=2}$, then the sky is falling.

T

F

tech #3 trivial proof $\cancel{\square} \rightarrow \triangle$

show that $\triangle$ is simply true (**ignore** $\square$)

if $\cancel{x=2}$, then $\boxed{3=3}$.

T

What if we can't do direct of $\square \to \triangle$?

(ex) n^2 is even $\to$ n is even

pf (by direct proof)

assume n^2 is even. $n^2 = 2 \cdot K$, K is an integer

$n = \sqrt{2K}$? $2(\text{int})$

~~let $n = 2l$~~, l is an integer
fallacy

n is even   show $n = 2 \cdot \text{integer}$

stuck!

try non-direct methods.

prove this directly

tech #4 $(\square \to \triangle) \equiv (\neg\triangle \to \neg\square)$

(ex) (n^2 is even $\to$ n is even) $\equiv$ (n is odd $\to$ n^2 is odd)

lemma

prove this (see above)

(ex) prove $\sqrt{2}$ is irrational

Integers: $\ldots, -2, -1, 0, 1, 2, \ldots$ $\mathbb{Z}$

rationals: $\frac{\text{int}}{\text{int}} = \frac{a}{b}$ such that a, b are ints
$b \neq 0$
a, b have no common factors