Math 321

Prove $\sqrt{2}$ is irrational — how?

1st lemma: If n^2 is even, then n is even

Pf (by contraposition)

prove contrapositive by a direct proof.

What are we proving?

Understand: #1 $\sqrt{x}$ mean:

question

$x^2 = x \cdot x$ vs $x^{1/2} \to ? \cdot ? = x$

$\sqrt{2}$ means $b \cdot b = 2$

#2 irrational

Natural, Whole numbers $1, 2, 3, \ldots$ / $0, 1, 2, 3, \ldots$

Integers $\ldots, -2, -1, 0, 1, 2, \ldots$

rational $\frac{a}{b}$ such that
1. a, b are integers
2. $b \neq 0$
3. a, b have no common factors

$\frac{2}{4}$

$\frac{5}{3} = 5\left(\frac{1}{3}\right)$

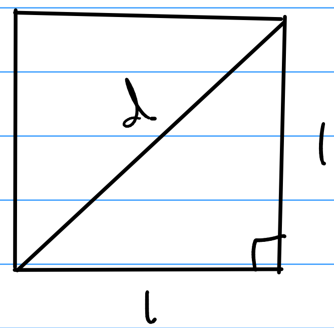
$\frac{1}{2}$

$\frac{a}{b} = a\left(\frac{1}{b}\right)$ $\quad \frac{\cancel{3}}{\cancel{6}} = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2} = \frac{\cancel{27}}{\cancel{4\pi}}$

→ actual measures

2.013 m

$$2 + \frac{1}{100} + \frac{3}{1000}$$



$$1^2 + 1^2 = d^2$$

$$d \cdot d = 2$$

$d = \sqrt{2}$ is irrational

$\boxed{\text{no}}$ $d = \frac{a}{b}$ exits so that $d \cdot d = 2$

Decimals

- rationals → decimal terminates or repeats
- irrationals → decimal doesn't terminate and doesn't repeat

0.121221222122221...

→ Reals : $\mathbb{R}$ Integers : $\mathbb{Z}$

Rationals : $\mathbb{Q}$

Prove: $\sqrt{2}$ is irrational

to hard?

goal show $\sqrt{2}$ is irrational $\equiv T$

same as $\neg(\sqrt{2}$ is irrational$) \equiv \neg T$

same as $\sqrt{2}$ is rational $\equiv F$

Pf (by contradiction)

$\sqrt{2}$ is rational. Means $\sqrt{2} = \frac{a}{b}$
and a,b are integers, $b \neq 0$, a,b have no common factors

So $\sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2}\,b = a \rightarrow (\sqrt{2}\,b)^2 = (a)^2$

$\boxed{2b^2 = a^2}$ so a^2 is even, now by lemma

$\boxed{a \text{ is even.}} \Longrightarrow a = 2 \cdot l$, l is an integer

$\Rightarrow 2b^2 = (2l)^2$

$2b^2 = 4l^2 \rightarrow b^2 = 2l^2$ So b^2 is even

now by lemma $\Rightarrow$ $\boxed{b \text{ is even}}$

$(a,b$ have no common factors$) \wedge (a,b$ have a common factor of $2) \equiv F$

So $\boxed{\sqrt{2} \text{ is irrational is always true}}$

∎

Conjecture #2 $\square \Longleftrightarrow \triangle$

$\equiv (\square \rightarrow \triangle) \wedge (\triangle \rightarrow \square)$