Math 321

Warnings on Proofs – typical mistakes

1. Fallacy of affirming the conclusion.
   (circular reasoning)
   (begging the question)
2. Other fallacies of reasoning.
3. Mistakes in "Math"
   - algebra, trig, calculus, operations, 'laws', etc

Conjecture #1 $(\square \to \triangle) \equiv T$

1. direct proof
2. contrapositive
3. contradiction

Conjecture #2 Show $\square \leftrightarrow \triangle$ is a tautology

Same as $\square \equiv \triangle$

two ways

1. $\square \equiv \text{step } 1 \equiv \text{step } 2 \equiv \dots \equiv \triangle$
2. $(\square \leftrightarrow \triangle) \equiv \underset{\text{case 1}}{(\square \to \triangle)} \wedge \underset{\text{case 2}}{(\triangle \to \square)}$

becomes 2 implications

Conjecture #3

$S_1 \equiv S_2 \equiv S_3 \equiv \cdots \equiv S_n$

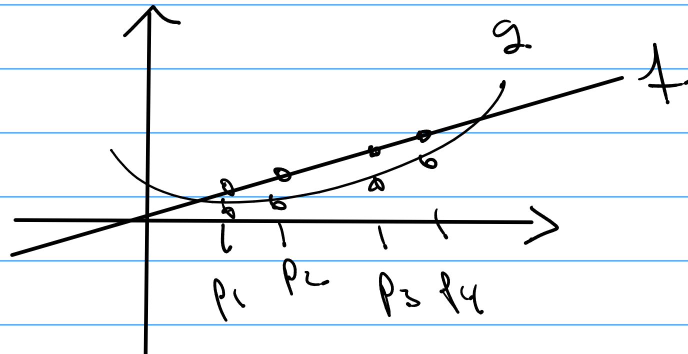
tech $(S_1 \to S_2) \wedge (S_2 \to S_3) \wedge \cdots \wedge (S_n \to S_1)$

(solve n-implications)

Conjecture #4

$(p_1 \vee p_2 \vee \cdots \vee p_n) \to c$

ex



$(x = p_1 \text{ or } p_2 \text{ or } p_3 \text{ or } p_4) \to f > g$

$\equiv \underbrace{(p_1 \to c)}_{\text{case \#1}} \wedge \underbrace{(p_2 \to c)}_{\text{case \#2}} \wedge \cdots \wedge \underbrace{(p_n \to c)}_{\text{case \#n}}$

(n-implications to prove)

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proof by cases

① cases are finite. case #1, #2, ..., #n

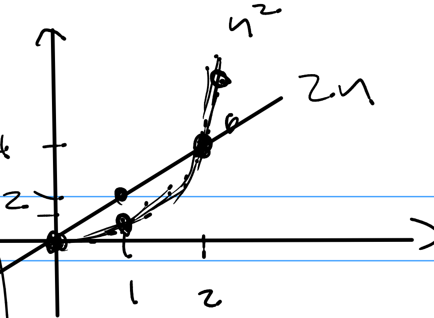
(called proof by exhaustion)

② cases are <u>not</u> finite

case #1, #2, #3, ...

(?)

(ex 6) n is a non-neg integer.



conjecture: if $n^2 > 2n$, then $n > 2$

pf ~~direct?~~ assume $n^2 > 2n$ $\overset{\text{algebra}}{\Rightarrow}$? $n > 2$

Contra position! show $\neg(n > 2) \rightarrow \neg(n^2 > 2n)$

non-neg. show $n \leq 2 \rightarrow n^2 \leq 2n$

show $(n=0 \text{ or } n=1 \text{ or } n=2) \rightarrow n^2 \leq 2n$

Case #1 if $n=0 \rightarrow n^2 \leq 2n$
let $n=0$ check $0^2 \leq 2 \cdot 0$ true

Case #2 if $n=1 \rightarrow n^2 \leq 2n$
let $n=1$ check $1^2 \leq 2 \cdot 1$ true

Case #3 if $n=2 \rightarrow n^2 \leq 2n$
let $n=2$ check $2^2 \leq 2 \cdot 2$ true

$\boxtimes$

Conjecture #5 $\exists x\, P(x)$ (existence proofs)

(ex) There is a square number beside a cubic number.

tech #1 constructive proof (find it)

$\square$ 1, 4, 9, 16, 25, 36, ...

cube: 1, 8, 27, 64, ...

8, 9

(ex) square number = cubic number.

proof $n = 64$

$64 = 8^2$ $64 = 4^3$

($1^2 = 1^3$ trivial)

$x^2 \wedge y^3 \rightarrow (\,)^{3 \cdot 2}$ $(\,)^6$

(tech #2) non-constructive.

You don't find the element for $\exists x P(x)$

we show it exists among a group of elements.

show $P(e_1) \oplus P(e_2) \equiv T$

(ex) $(\text{irrational})^{\text{irrational}}$ is rational

(ex) $(\text{irrational})^{\text{rational}}$ is integer

$(\sqrt{2})^2 = 2$ constructive

1) pf $(\sqrt{2})^{\sqrt{2}}$ is a real number

case 1 $(\sqrt{2})^{\sqrt{2}}$ is rational done

case 2 $(\sqrt{2})^{\sqrt{2}}$ is irrational

non-constructive

check $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^2 = 2$ rational

done

Open problems / open conjectures

3x+1

$3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$