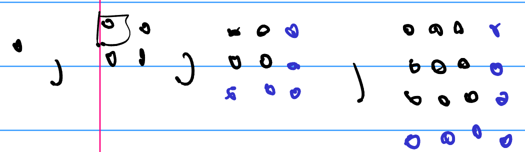
Math 321

Q's / $\boxed{1.8 \# 9}$ 100 consecutive positive integers

$p, p+1, p+2, \ldots, p+99$

None of which are squares.



$1, 4, 9, 16, 25$

$+3 \quad +5 \quad +7 \quad +9$

$4, \boxed{5, 6, 7, 8,} 9$

$(9-4) = 5$

$n^2 \longrightarrow (n+1)^2$

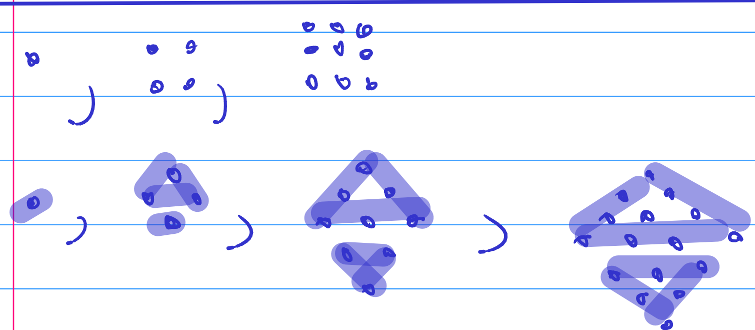
Solve $(n+1)^2 - n^2 - 1 \geq 100$

$\cancel{n^2} + 2n + \cancel{1} - \cancel{n^2} - \cancel{1} \geq 100$

$2n \geq 100$

$n \geq 50$

$50^2 \; \boxed{} \; 51^2$



1.8) 13

$(\text{rational})^{(\text{irrational})} = \text{irrational}$

Existence?

Conjecture: $2^{\sqrt{2}}$ is irrational.

Guess

(contradiction)

Pf assume $2^{\sqrt{2}}$ is rational.

$$2^{\sqrt{2}} = \frac{a}{b}$$ where a, b are ints, $b \neq 0$, no common factors

Stuck

$$\ln\left(2^{\sqrt{2}}\right) = \ln\left(\frac{a}{b}\right)$$

$$\sqrt{2}\ln 2 = \ln(a/b) = \ln(a) - \ln(b)$$ Stuck

Consider $2^{\sqrt{2}}$ → if irrational **done**

→ if $2^{\sqrt{2}}$ is rational

$$\left(2^{\sqrt{2}}\right)^{\frac{\sqrt{2}}{4}} = \sim = \sqrt{2} = \boxed{2^{1/2}}$$

$\left(\sqrt{2}^{\sqrt{2}}\right)$ → rational done

→ $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ etc. done

Consider:
$$\left(2^{\sqrt{2}}\right)^{\frac{1}{4}\sqrt{2}} = 2^{1/2} = \sqrt{2} \text{ irrational}$$

$\frac{1}{4}\sqrt{2}$ — irrational

$2^{\sqrt{2}}$ — rational

Lemma: $\frac{1}{4}\sqrt{2}$ is irrational (rational)(irrational)

$$\text{ir} = \frac{c}{d} \cdot \frac{b}{a}$$

Exam 1 11 probs @ 10 pts each

100 pts = 100%

1.1 / 1.2 propositional logic (no quantification)

2 probs ① Give the truth table everyone should know.

② eng → symbols , symbols → english

1.3 logical equiv.

3 probs ① show □ ≡ Δ by a truth table

(show

	□ ⇔ Δ
	T
	T
	⋮
	T

② show □ ≡ Δ by discussion

③ use logical equiv. to rewrite a compound proposition.

ex show ($\underbrace{\sim\sim\sim}_{\text{compound prop}}$) is a tautology by using log. equiv.

(compound prop) ≡ step 1 ≡ step 2 ≡ .. ≡ T

(1.4/1.5) Quantification

2 probs: (1) symbols → english

P(r,s): " ... "
Q(s,r,t): " ... "

$\forall x \exists y \left[P(x,y) \rightarrow \exists z\, Q(x,y,z) \right]$

U.D.

(2) english → symbols

(1.6) Rules of Inference

(1 prob) 1a) why is an argument not valid.

1b) given premises → valid conclusions?

(1.7/1.8) Proofs

(3 probs) (1) $\sqrt{2}$ is irrational

pf: Step 1: lemma n^2 even → n even

Step 2 (by contradiction)

(2) proof of cases

(3) Existence proof.