Math 321

Math = Toys + Rules

Propositional Logic — Propositions — $\wedge, \vee, \oplus, \rightarrow$, etc, $\equiv$, rules of inference, quantification

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(Naive) Set theory — Toys = Sets

rules = ??

Set: unordered collection of objects
elements.

Notation: lowercase letter : element
uppercase letter : set

$e \in S$: "e is an element of set S"

$e \notin S$: "e is not an element of set S"

Represent Sets

(i) Roster or List method.

ex $S = \{1, 2, 4, 7, 11\}$

$S = \{1, 2, 3, \ldots, 51\}$

Note: Set brackets $\{\ \}$ order does not matter

$(\ )$ order does matter

ex $S = \{1, a, \Delta, ☺, \{1,2\}\}$

B) Computer rep. of a set (List of 0s 1s)

i) Universe of Discourse (all possible elements)

ex $U = \{1,2,3,4,5,6,7,8,9\}$

$E = \{2,4,6,8\}$

$O = \{1,3,5,7,9\}$

Computer: $U = (1,2,3,4,5,6,7,8,9)$

rep

U:	1	1	1	1	1	1	1	1	1
E:	0	1	0	1	0	1	0	1	0
O:	1	0	1	0	1	0	1	0	1

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ii) Set Builder notation

$S = \{ e \mid P(e) \}$

$\mid$: such that

e : element form

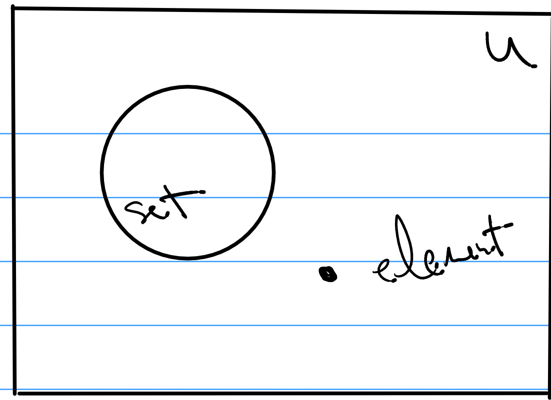
$P(e)$: propositional function.

ex $U = \{1, 2, \ldots, 100\}$

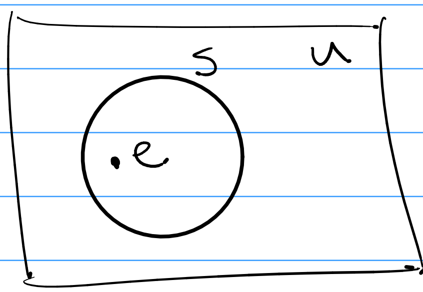
$S = \{ x \mid x \in U \wedge x = 3 \cdot \text{integer} \}$

$S = \{3, 6, 9, 12, \ldots, 99\}$

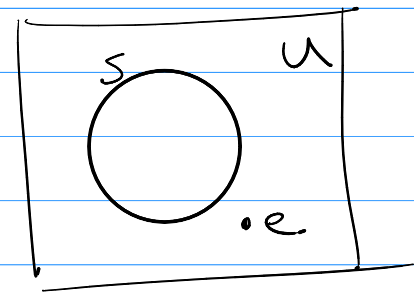
③ Venn Diagrams



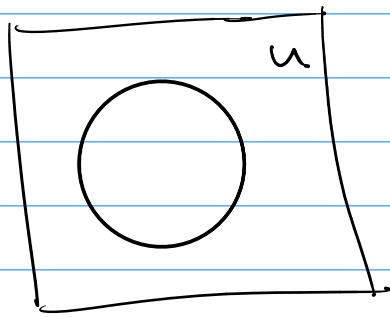
$e \in S$



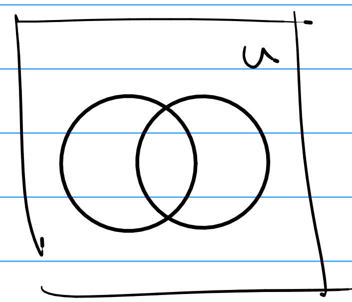
$e \notin S$



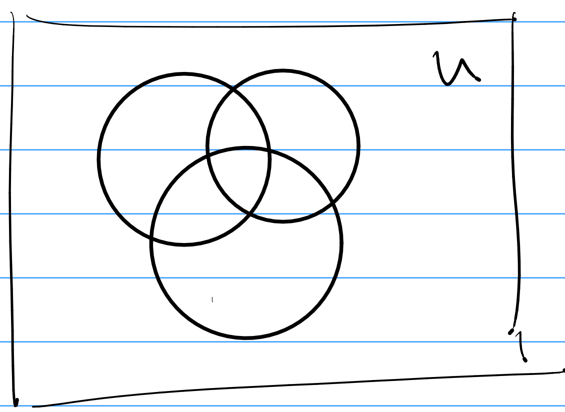
1 set



2 sets



3 sets



Sets to Know

U : set of everything in discussion

$\emptyset$: set with nothing in it (empty set)

$\emptyset = \{ \}$

Number Sets

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ (naturals, non-negative integers)

ints. $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

rationals $\mathbb{Q} = \{ \frac{a}{b} \mid a, b \in \mathbb{Z} \land b \neq 0 \land a, b \text{ have no common factors} \}$

$\mathbb{R}$: real numbers ($\mathbb{Q}$ and the irrationals)

$\mathbb{R}^+$ (positive reals)

$\mathbb{C} = \{ a + bi \mid i = \sqrt{-1} \land a, b \in \mathbb{R} \}$

Rules?

Comparisons.

① **Subset** $A \subseteq B$

$\Leftrightarrow \quad \forall e (e \in A \rightarrow e \in B)$

② **Proper subset** $A \subset B$

$\Leftrightarrow \quad \forall e (e \in A \rightarrow e \in B)$

and $\exists x (x \in B \land x \notin A)$

(3) equal $A = B$

$$\forall e\,((e \in A \to e \in B) \land (e \in B \to e \in A))$$

$$\equiv \forall e\,(e \in A \leftrightarrow e \in B)$$

Problem: distinct elements

$$\left.\begin{array}{l} A = \{1, 2, 2, 3, 4\} \\ B = \{1, 2, 3, 3, 3, 3, 4, 4\} \end{array}\right\} A = B$$

So... usually only list distinct elements..

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4\}$

(4) Cardinality. (counts how many distinct elements)

$|S|$ = S's cardinality.

$|\{☺, \Delta, \{1, 3\}, \{\}\}| = 4$

$|\{\}| = 0$

$|\emptyset| = 0$

$|\{1,2,\ldots,51\}| = \boxed{51}$ a finite set

$|\mathbb{Z}| =$ not finite $=$ infinite