Math 321

Naive Set Theory (continued)

2.1 3a) $A \subseteq B$?, $B \subseteq A$?, neither?

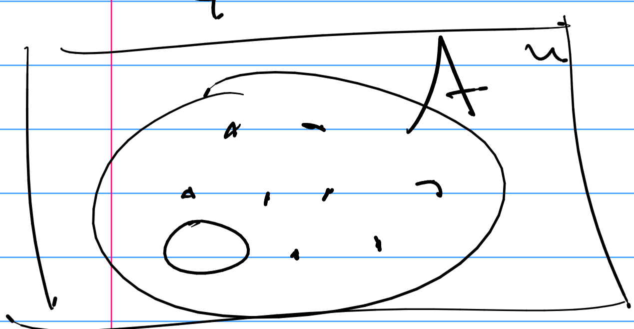
A = { airline flights from NYC to New Delhi }

B = { non-stop from NYC to New Delhi }

$B \subseteq A \quad \forall f\,(f \in B \to f \in A)$

$A \subseteq A$ b/c $\forall e\,(e \in A \to e \in A)$

$\emptyset \subseteq A$ b/c $\forall e\,(e \in \emptyset \to e \in A)$



$\equiv \forall e\,(F \to e \in A)$

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7) is $2 \in S$?

Ex) $S = \{ e \mid e \text{ is even} \} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$

$2 \in S$? $\boxed{\text{Yes}}$

7b) $S = \{ \{2\}, \{\{2\}\} \}$ $\quad 2 \notin S$

7c) $S = \{ 2, \{2\} \}$ $\quad 2 \in S$

1(e) $\phi \subseteq \{x\}$ True

1(f) $\phi \in \{x\}$ False

(ex) $\phi \subseteq \{\phi, \ddot\smile, \square\}$ True

$\phi \in \{\phi, \ddot\smile, \square\}$ True

Operations on Sets.

(1) Power Set $P(S) = \{$ all possible subsets of a set S $\}$

$P(S) = \{ \phi$ (no elements)
(one element subsets)
(two element subsets)
$\vdots$
S (all elements) $\}$

(ex) $P(\{1, \square, \ddot\smile\}) = \{ \phi, \{1\}, \{\square\}, \{\ddot\smile\}, \{1, \square\}, \{1, \ddot\smile\}, \{\square, \ddot\smile\}, \{1, \square, \ddot\smile\} \}$

$|P(\{1, \square, \ddot\smile\})| = 8 = 2^3$

$|P(S)| = 2^{|S|}$

$|P(\phi)| = 2^{|\phi|} = 2^0 = 1$

$$P(\emptyset) = \{\emptyset\}$$

$$|P(P(\emptyset))| = 2^{|P(\emptyset)|} = 2^1 = 2$$

$$P(P(\emptyset)) = P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$$

$$P(P(\{\})) = \{\{\}, \{\{\}\}\}$$

Cross Product / Cartesian Product

$$A \times B = \{ \underbrace{(a,b)}_{\text{ordered pair}} \mid a \in A \wedge b \in B \}$$

$$A_1 \times A_2 \times \dots \times A_n = \{ \underbrace{(a_1, a_2, \dots, a_n)}_{\text{ordered n-tuple}} \mid a_i \in A_i \}$$

$$|A_1 \times A_2 \times \dots \times A_n| = |A_1||A_2| \dots |A_n|$$

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25 Conjecture: $P(A) \subseteq P(B) \leftrightarrow A \subseteq B$

Pf: $\boxed{\text{Case 1}}$ if $A \subseteq B \to \boxed{P(A) \subseteq P(B)}$

$\boxed{\text{Case 2}}$ if $P(A) \subseteq P(B) \to A \subseteq B$

$\boxed{\text{Idea}}$ $\boxed{\text{Case 1}}$ $\boxed{\text{Direct}}$ assume $A \subseteq B$

so $\forall e\,(e \in A \to e \in B)$

goal? Show $\boxed{P(A) \subseteq P(B)}$

$$\equiv \forall S\,(S \in P(A) \to S \in P(B))$$

$$\equiv \forall S\,(\boxed{S \subseteq A} \to S \subseteq B)$$

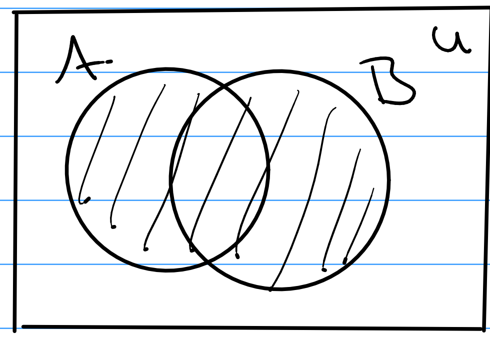
$$\forall S\,(\forall e\,(e \in S \to e \in A) \to \forall e\,(e \in S \to e \in B))$$

Other ops

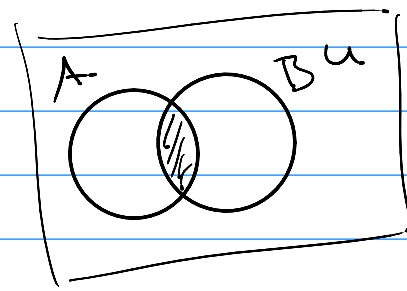
closed type

① $A \cup B = \{e \mid e \in A \vee e \in B\}$

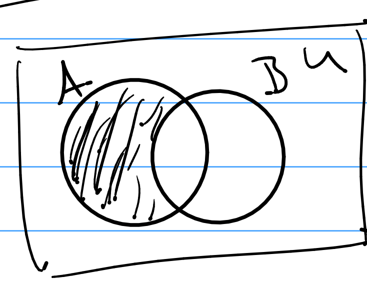
which



② $A \cap B = \{e \mid e \in A \wedge e \in B\}$

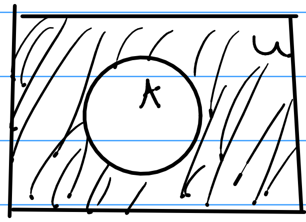


③ $A - B = \{e \mid e \in A \wedge e \notin B\}$



④ $A \oplus B = \{e \mid e \in A \oplus e \in B\}$

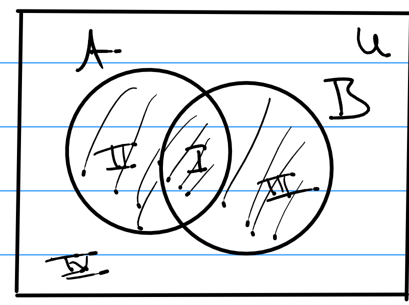
⑤ $\overline{A} = U - A = \{ e \mid e \in U \wedge e \notin A \}$



$= \{ e \mid e \notin A \}$

Membership Tables

A	B	A ∪ B
1	1	1
1	0	1
0	1	1
0	0	0



Set Identities

$A \cup (B \wedge C) = (A \cup B) \wedge (A \cup C)$

$A \cup (B \wedge C) = \{ e \mid e \in A \vee (e \in B \wedge e \in C) \}$

$= \{ e \mid (e \in A \vee e \in B) \wedge (e \in A \vee e \in C) \}$

$= (A \cup D) \wedge (A \cup C)$