Math 321

Set ops $A \cup B = \{e \mid e \in A \vee e \in B\}$

$A \cap B = \{e \mid e \in A \wedge e \in B\}$

$A - B = \{e \mid e \in A \wedge e \notin B\}$

$A \oplus B = \{e \mid e \in A \oplus e \in B\}$

$\overline{A} = \{e \mid e \notin A\}$

Show: $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

① (Logic) $\overline{A \cap B} = \{e \mid e \notin (A \cap B)\}$

$= \{e \mid \neg (e \in (A \cap B))\}$

$= \{e \mid \neg (e \in A \wedge e \in B)\}$

$\boxed{=} \{e \mid \neg(e \in A) \vee \neg(e \in B)\}$

$= \{e \mid e \in \overline{A} \vee e \in \overline{B}\}$

$= \{e \mid e \in (\overline{A} \cup \overline{B})\}$

$= \overline{A} \cup \overline{B}$

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② $S_1 = S_2$ show $\underbrace{S_1 \subseteq S_2}_{\text{case 1}}$, $\underbrace{S_2 \subseteq S_1}_{\text{case 2}}$

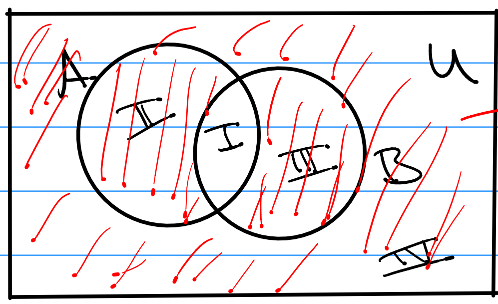
case 1 $S_1 \subseteq S_2$ show $e \in S_1 \to e \in S_2$

$\boxed{\text{Case 2}}$ $S_2 \subseteq S_1$ <u>show</u> $e \in S_2 \to e \in S_1$

(3) <u>Membership tables.</u> $\overline{A \cap B} = \bar{A} \cup \bar{B}$

row	A	B	$A \cap B$	$\overline{A \cap B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$
I	1	1	1	0	0	0	0
II	1	0	0	1	0	1	1
III	0	1	0	1	1	0	1
IV	0	0	0	1	1	1	1

<u>Same regions.</u>



$\overline{A \cap B}$

<u>Extra Stuff:</u>

(1) $A_1 \cup A_2 \cup A_3 \cup \dots = \bigcup_{i=1}^{\infty} A_i$

(2) $A_1 \cap A_2 \cap A_3 \cap \dots = \bigcap_{i=1}^{\infty} A_i$

(Ex) $A \cup \overline{(B \cap C)} = \overline{\bar{A} \cap (\bar{B} \cap C)} = \boxed{\overline{\bar{A} \cap \bar{B} \cap C}}$

(Ex) $A \cup \overline{(\bar{A} \cap C)} = A \cup (A \cup \bar{C}) = A \cup A \cup \bar{C}$

$= \boxed{A \cup \bar{C}}$

2.3 Functions

Cartesian Product :

$$A_1 \times A_2 \times \cdots \times A_n = \{ \underbrace{(a_1, a_2, \ldots, a_n)}_{\text{all possible } n\text{-tuples}} \mid \forall i \ a_i \in A_i \}$$

ex (students) X (phone type) X (shoe type)

$= \{ (s, p, shoe) \mid \qquad \}$

Subset of this cartesian product would be a relationship (n-ary relation)

Cross Product $A \times B = \{ (a,b) \mid a \in A \land b \in B \}$

Subset of this is a binary relation or just relation

Function $f : A \to B$ (subset of $A \times B$)

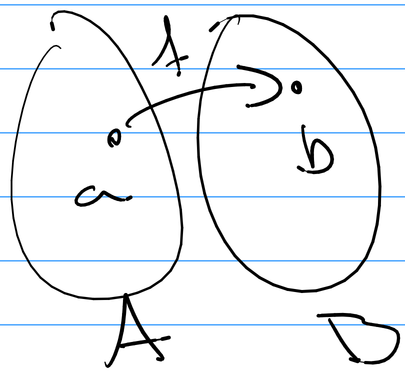
assigns exactly one $b \in B$ for all $a \in A$.

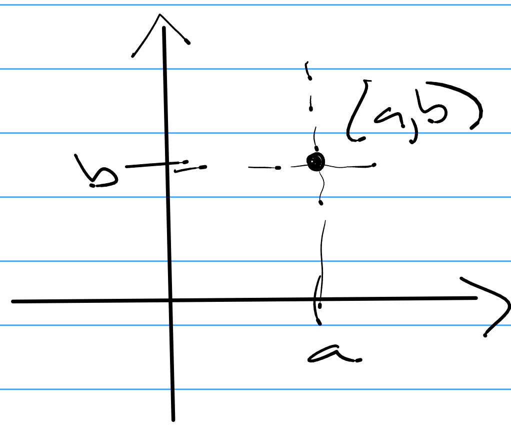
Notation / Visualize

$$f: A \to B$$
domain — codomain

$\underline{(a,b) \in f}$ — a is a preimage of b

$f(a) = b$ — b is an image of a





$f: \mathbb{R} \to \mathbb{R}$ $\quad a \to b$

$\boxed{\text{ops}}$

$$(f+g)(x) = f(x) + g(x)$$
$$(fg)(x) = f(x) \cdot g(x)$$
$$(f \circ g)(x) = f(g(x))$$

Identities / Inverses (of any Math)

Identities: does nothing under some operator

Inverses: make an identity from an element under some operator

logic: $p \land T \equiv p \qquad \cancel{p \land \quad} \equiv T$

arith

$3 \cdot 1 = 3$

$x \cdot \frac{1}{x} = 1$

$x \neq 0$