Math 321

Functions $\quad f \circ g$

Identity (Function) under composition

$$I(x) = x$$

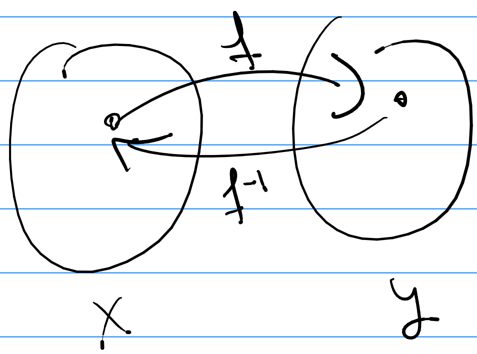
$$(f \circ I)(x) = f(I(x)) = f(x)$$

$$(I \circ f)(x) = I(f(x)) = f(x)$$

Inverse $\quad (f \circ f^{-1})(x) = x$

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

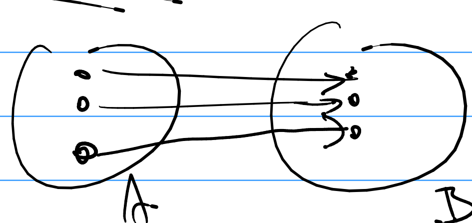


check if f, g are inverses

$$\left.\begin{aligned} f(g(x)) &= x \\ g(f(x)) &= x \end{aligned}\right\} \text{ say } g = f^{-1}$$

When does f have an inverse?

$f: A \to B$



f is onto when $\boxed{\text{range} = \text{codomain}}$

f is one-to-one

$$\forall a \forall b \, (f(a) = f(b) \rightarrow a = b)$$

$$\equiv \forall a \forall b \, (a \neq b \rightarrow f(a) \neq f(b))$$

f^{-1} exists if $\boxed{f \text{ is one-to-one } \underline{\underline{\text{and}}} \text{ onto (bijection)}}$

Special functions

$f: \mathbb{R} \rightarrow \mathbb{Z}$

① $\lceil x \rceil$ ceiling function

$$\lceil x \rceil = \begin{cases} x & \text{if } x \text{ is an int.} \\ y & x \text{ is not an int, } y \text{ is an int} \\ & y \text{ is 1st int to the right of } x \end{cases}$$

② $\lfloor x \rfloor$ floor function

③ round (x)

$f: \mathbb{N} \rightarrow \mathbb{N}$

$n! = n(n-1)(n-2)\cdots(1)$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

Def: $0! = 1$

$\boxed{2.4}$ **Sequences** $\left(f : \boxed{\mathbb{N}} \to A\right)$

$0 \xrightarrow{f} a$

$1 \to b$

$2 \to b$

$3 \to c$

$\vdots \qquad \vdots$

$a, b, b, c, \ldots$ ← **sequence**

$0 \; 1 \; 2 \; 3$

Notation: $f(n) = 3n + n^2 \qquad n = 0, 1, 2, \ldots$

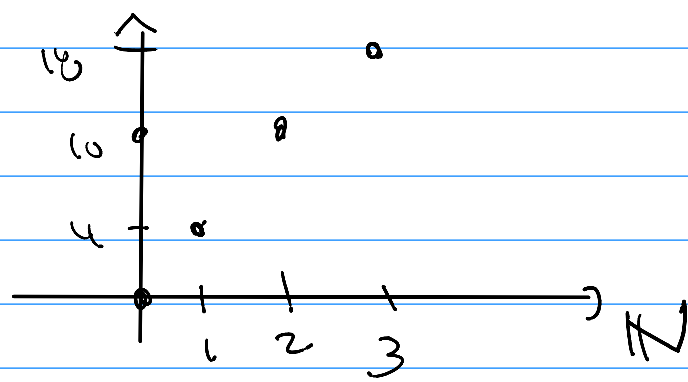
$(0, 0)$

$(1, 4)$

$(2, 10)$

$(3, 18)$

$\vdots$



Seq: $0, 4, 10, 18, \ldots$

$f(n) = 3n + n^2$ **rather use:** $a_n = 3n + n^2$

$\{3n + n^2\}$ $n = 0, 1, 2, 3,$

$\boxed{\{a_n\} \quad n = 0, 1, 2, \ldots \quad \text{becom} \quad a_0, a_1, a_2, \ldots}$

Sequences to know

(1) $\{a+dn\}$ $n=0,1,2,\ldots$ arithmetic

$a, a+d, a+2d, a+3d, \ldots$

(2) $\{ar^n\}$ $n=0,1,2,\ldots$ geometric

$a, ar, ar^2, ar^3, \ldots$

(3) $\{n^2\}$ $n=0,1,2,\ldots$

$0, 1, 4, 9, \ldots$

(4) $\{n^3\}$ $n=0,1,2,\ldots$

$0, 1, 8, 27, \ldots$

(5) $\{2^n\}$ $n=0,1,2,\ldots$

(6) $\{3^n\}$ $n=0,1,2,\ldots$

(7) $\{n!\}$ $n=0,1,2,\ldots$

$\{a_n = 2^n\}$ $n=0,1,2,\ldots$

$a_0 = 1, \ a_1 = 2, \ a_2 = 4, \ a_3 = 8, \ldots$

$1, 2, 4, 8, 16, \ldots$