Math 321

Q5/ $A \cup (A \cap B) = A$

Show $S_1 = S_2$ ✓

3 ways
① $S_1 \subseteq S_2 \wedge S_2 \subseteq S_1$
② Set builder with logic
$S_1 = \{e \mid \text{prop. function}\} = \cdots = S_2$
③ Membership table

Show: case 1 $e \in A \rightarrow e \in A \cup (A \cap B)$

case 2 $e \in A \cup (A \cap B) \rightarrow e \in A$

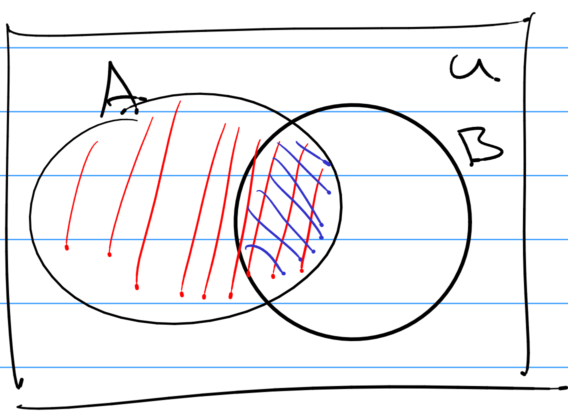
Set builder

$A \cup (A \cap B) = \{e \mid e \in (A \cup (A \cap B))\}$

$= \{e \mid e \in A \vee e \in (A \cap B)\}$

$= \{e \mid e \in A \vee (e \in A \wedge e \in B)\}$

$= \{e \mid e \in A\} = A$



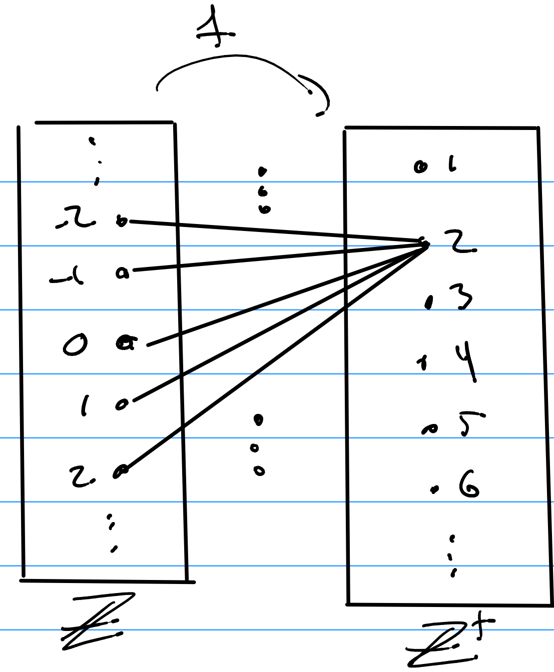
$A \cup (A \cap B) = A$

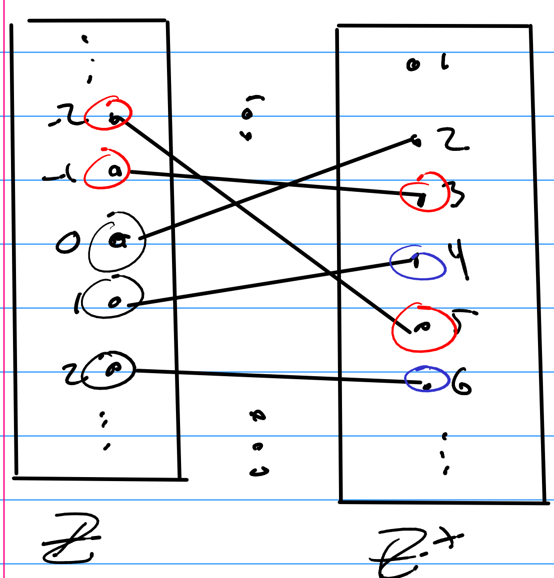
Functions: $f: \mathbb{Z} \to \mathbb{Z}^+$

(ex) not one-to-one and not onto

words: "send everyone to 2"

algebra: $f(x) = 2$





1) one-to-one
not onto

words: send negatives to odds starting at 3.
set non-negs to evens.

$$f(x) = \begin{cases} 2|x|+1 & x < 0 \\ 2(x+1) & x \geq 0 \end{cases}$$

Seq

$\{a_n\}$ $n = 0, 1, 2, \ldots$ — Rule/function

$a_0, a_1, a_2, \ldots$ — seq.

closed form: (ex) $a_n = 2n + n^2$ $n = 0, 1, 2, \ldots$

$0, 3, 8, 15, \ldots$

open form / recurrence relation / inductive form

Basis: start element(s)

Rule make new values from old.

open / recurrence inductive

ex. Basis

$a_0 = 1$

$a_n = 2a_{n-1} + 3$

Seq: 1, 5, 13, 29, ...

n = 1, 2, 3, ...

$a_1 = 2a_0 + 3 = 2(1) + 3 = 5$

$a_2 = 2a_1 + 3 = 13$

$a_3 = 2a_2 + 3 = 29$

Fibonacci Seq: 0, 1, 1, 2, 3, 5, 8, 13, ...

$a_0 = 0, \; a_1 = 1$ $f_0 = 0, \; f_1 = 1$

$a_n = a_{n-1} + a_{n-2}$ $f_n = f_{n-1} + f_{n-2}$ n = 2, 3, ...

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open seq rule → closed rule

forward reasoning / backward reasoning

a_0 $a_n =$

$a_1 =$ open rule $a_{n-1} =$ open rule

$a_2 =$ open rule $a_{n-2} =$ open rule

⋮ ⋮

$a_n =$ open rule ← see a pattern → $a_0 =$

(ex) $a_0 = 1 \qquad a_n = 2a_{n-1} + 3$

forward

$a_0 = 1$

$a_1 = 2(1) + 3$

$a_2 = 2(2+3) + 3 = 2^2 + 2\cdot 3 + 3$

$a_3 = 2(2^2 + 2\cdot 3 + 3) + 3 = 2^3 + 2^2\cdot 3 + 2\cdot 3 + 3$

$a_4 = 2(2^3 + 2^2\cdot 3 + 2\cdot 3 + 3) = 2^4 + 2^3\cdot 3 + 2^2\cdot 3 + 2\cdot 3 + 3$

$$a_n = 2^n + \left| 2^{n-1}\cdot 3 + 2^{n-2}\cdot 3 + \dots + 2^1\cdot 3 + 2^0\, 3 \right|$$

$$a_n = 2^n + 3\left(2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0\right)$$

add of seq. (pause)

Summation

Sums $\displaystyle\sum_{i=n}^{m} a_i = a_n + a_{n+1} + \dots + a_m$

ex $\displaystyle\sum_{i=2}^{7} 2i+1 = (2\cdot 2+1) + (2\cdot 3+1) + \dots + (2\cdot 7+1)$

$\displaystyle\sum_{i=1}^{n} 1 = 1 + 1 + 1 + \dots + 1 = n$

1 to n is n sums

$\displaystyle\sum_{i=1}^{n} c = n\cdot c$

$$\sum_{i=1}^{n} i = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=2}^{7} 2i+1 = 2\left(\sum_{i=1}^{7} i - 1\right) + \sum_{i=2}^{7} 1$$

$$= 2\left[\frac{7(8)}{2} - 1\right] + 6$$

$$= 7\cdot 8 - 2 + 6 = \boxed{60}$$

Series telescoping $\sum_{i=1}^{n} a_{i+1} - a_i$

ex $\sum_{i=1}^{10} \frac{1}{i} - \frac{1}{i+1} =$

$$\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$$

$$+ \dots + \left(\frac{1}{9} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{11}\right)$$

$$= \boxed{1 - \frac{1}{11}}$$

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ex

$1 = 0.99999\ldots$

$\frac{1}{4} = 0.25 = 0.2499999\ldots$

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ex

$.999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots = S$

$10S = 9 + \left[\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots\right]$

$10S = 9 + S$

$9S = 9$

$\boxed{S = 1}$

$$\sum_{k=0}^{n} ar^k = \begin{cases} a\left(\frac{r^{n+1}-1}{r-1}\right) & \text{if } r \neq 1 \\ a(n+1) & \text{if } r = 1 \end{cases}$$

$$3 \cdot 2^{n-1} + 3 \cdot 2^{n-2} + \cdots + 3 \cdot 2^1 + 3 \cdot 2^0 = 3\left(\frac{2^n - 1}{2-1}\right)$$

$$= 3(2^n - 1)$$

(back from prev.)

$$a_n = 2^n + 3(2^n - 1) = 4 \cdot 2^n - 3$$