Math 321

Q's 17c) $a_n = a_{n-1} + n$ $\quad n = 1, 2, 3 \dots$ $\quad a_0 = 1$ $\quad a_1 = a_0 + 1$

$a_2 = a_1 + 2$

$a_3 = a_2 + 3$

$\vdots$

seq: $1, 2, 4, 7, 11, 16, \dots$ ($+1$, $+2$, $+3$, $+4$, $+5$)

$$a_n = \underbrace{a_{n-1}}_{a_{n-2}+(n-1)} + n = \underbrace{a_{n-2}}_{a_{n-3}+(n-2)} + (n-1) + n = a_{n-3} + (n-2) + (n-1) + n$$

So $a_n = \underbrace{a_0}_{1} + \underbrace{1 + 2 + 3 + 4 + \dots + n}_{\frac{n(n+1)}{2}}$

$n - n$

$$\boxed{a_n = \frac{n(n+1)}{2} + 1}$$

33c) $$\sum_{i=1}^{3}\left(\sum_{j=0}^{2} i\right) = \sum_{i=1}^{3} i \underbrace{\sum_{j=0}^{2} 1}_{3 = 1+1+1} = \sum_{i=1}^{3} 3i$$

$$= 3 \sum_{i=1}^{3} i = 3\left(\frac{3 \cdot 4}{2}\right) = \boxed{18}$$

$(1+2+3)$

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35) $$\sum_{j=1}^{n} (a_j - a_{j-1}) = \underbrace{(\cancel{a_1} - a_0)}_{j=1} + \underbrace{(\cancel{a_2} - \cancel{a_1})}_{j=2} + \underbrace{(\cancel{a_3} - \cancel{a_2})}_{j=3}$$

$$\dots + \underbrace{(\cancel{a_{n-1}} - \cancel{a_{n-2}})}_{j=(n-1)} + \underbrace{(a_n - \cancel{a_{n-1}})}_{j=n} = \boxed{a_n - a_0}$$

Cardinality

$|S|$ (notation)

① $|S| = n$ (is a finite number)

② $|\mathbb{Z}^+|$ is not finite (infinite)

$|\mathbb{Z}^+| = \aleph_0$

Def: ① $|S_1| = |S_2|$ when there is a bijection from S_1 to S_2

② $|S_1| \leq |S_2|$ if there is a one-to-one function from S_1 to S_2.

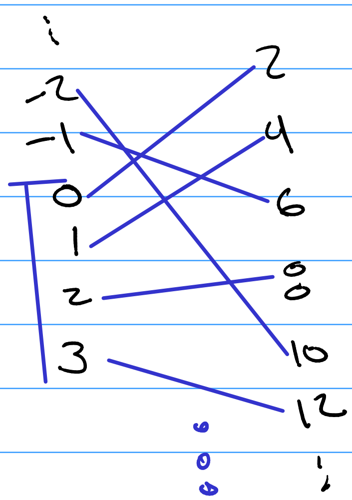
③ $|S_1| \leq |S_2| \land |S_1| \neq |S_2| \rightarrow |S_1| < |S_2|$

Ex

evens $= \{2, 4, 6, 8, 10, \ldots\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$|\text{evens}| = |\mathbb{Z}|$



Hilbert's Grand Hotel

$$\mathbb{Z}^+ \quad 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ \dots$$

$$\begin{array}{ll}
 & \quad 2 \quad\;\; 3 \quad\;\; 4 \quad\;\; 5 \\
L_1 & M_{11},\ M_{12},\ M_{13},\ M_{14},\ M_{15},\ \dots \\
L_2 & M_{21},\ M_{22},\ M_{23},\ M_{24},\ M_{25},\ \dots \\
L_3 & M_{31},\ M_{32},\ M_{33},\ M_{34},\ M_{35},\ \dots \\
L_4 & M_{41},\ M_{42},\ M_{43},\ M_{44},\ M_{45},\ \dots \\
\vdots & \vdots \quad\; \vdots \quad\; \vdots \quad\; \vdots \quad\; \vdots
\end{array}$$

$$\begin{array}{llllll}
0 & & & & & \\
\pm\frac{1}{1} & \pm\frac{1}{2} & \pm\frac{1}{3} & \pm\frac{1}{4} & \pm\frac{1}{5} & \dots \\
\pm\frac{2}{1} & \cancel{\pm\frac{2}{2}} & \pm\frac{2}{3} & \cancel{\pm\frac{2}{4}} & \pm\frac{2}{5} & \dots \\
\pm\frac{3}{1} & \pm\frac{3}{2} & \cancel{\pm\frac{3}{3}} & \pm\frac{3}{4} & \pm\frac{3}{5} & \dots \\
\pm\frac{4}{1} & \cancel{\pm\frac{4}{2}} & \pm\frac{4}{3} & \pm\frac{4}{4} & \pm\frac{4}{5} & \dots \\
\vdots & \vdots & \vdots & \vdots & & \ddots
\end{array}$$

all $\frac{a}{b}$ such that

$a, b \in \mathbb{Z}$, $b \neq 0$

$|\mathbb{Q}| = \aleph_0$

$$\boxed{\begin{array}{l} |S| = n \text{ finite} \\ |S| = \aleph_0 \text{ is called Countable} \end{array}}$$

$|\mathbb{R}| = ?$

$\mathbb{R}$ only consider $(0,1)$

$r = 0.$ decimals

assume these are countable...

$d_{ij} = \{0,1,2,\ldots,9\}$

$1 \to r_1 = 0.d_{11}d_{12}d_{13}d_{14}\ldots$

$2 \to r_2 = 0.d_{21}d_{22}d_{23}d_{24}\ldots$

$3 \to r_3 = 0.d_{31}d_{32}d_{33}d_{34}\ldots$

$4 \to r_4 = 0.d_{41}d_{42}d_{43}d_{44}$

$\vdots$

$r = 0.57557\ldots$

b/c $\overline{9}$ are also term. decimals

$0.\cancel{249999}\ldots = 0.25$

no $\overline{9}$ allowed in list.

$\to r_i$ is uniq. decimal

see video !!