Math 321

$\mathbb{Q}$ is countable $\qquad \mathbb{Q} = \{ \frac{a}{b} \mid a,b \in \mathbb{Z},\ b \neq 0,\ \text{no common factors} \}$

Idea:

	d=1	d=2	d=3	d=4
1: 0				
	2,3: $\pm\frac{1}{1}$	6,7: $\pm\frac{2}{1}$	$\pm\frac{3}{1}$	$\cdots$
	4,5: $\pm\frac{1}{2}$	$\cancel{\pm\frac{2}{2}}$ SKIP	$\pm\frac{3}{2}$	$\cdots$
	8,9: $\pm\frac{1}{3}$	$\pm\frac{2}{3}$	$\pm\frac{3}{3}$	$\cdots$
	$\vdots$	$\vdots$	$\vdots$	

table is $\left\{ \frac{a}{b} \;\middle|\; a,b \in \mathbb{Z},\ b \neq 0 \right\}$

d number is $a+b$ except 0 is $d=1$

$\mathbb{R}$ is uncountable

Pf (by contradiction; assume $\boxed{\mathbb{R} \text{ countable}}$, show $\equiv$ $\boxed{\text{Always False}}$)

Consider $\mathbb{R}$ in interval from 0 to 1.

know any $r \in (0,1)$ is $r = 0.d_1 d_2 d_3 \ldots$

$d_i \in \{0,1,2,\ldots,9\}$

assume countable then we have the seq.

$r_1 = 0.d_{11} d_{12} d_{13} \ldots$

$r_2 = 0.d_{21} d_{22} d_{23} \ldots$

$r_3 = 0.d_{31} d_{32} d_{33} \ldots$

$\vdots$

one-to-one and $\boxed{\text{onto}}$

except b/c any term. decimal has two representations

ex $0.13999\ldots = 0.14$

exclude all $\overline{9}$ versions

So each r_i has a unique representation.

Consider a special number I will create...

$$r^* = 0.d_1 d_2 d_3 d_4 \ldots$$

b/c uniq. rep.

① Pick d_1 so that $d_1 \neq d_{11}$ $\rightarrow$ $r^* \neq r_1$

② Pick d_2 so that $d_2 \neq d_{22}$ $\rightarrow$ $r^* \neq r_2$

③ Pick d_3 so that $d_3 \neq d_{33}$ $\rightarrow$ $r^* \neq r_3$

⋮

$$\forall i \quad d_i \neq d_{ii} \rightarrow r^* \neq r_i$$

r^* is in $(0,1)$ so our list ← function

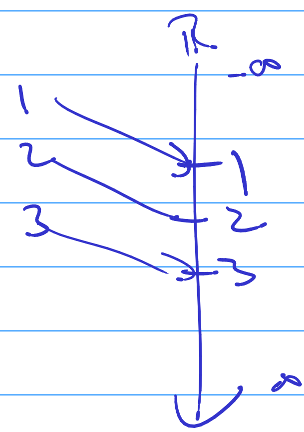
is onto and not onto $\equiv$ F

$\therefore$ $\mathbb{R}$ is uncountable.

So ① $|\mathbb{R}| \neq |\mathbb{Z}^+| = \aleph_0$

② $\mathbb{Z}^+ \subseteq \mathbb{R}$

$|\mathbb{Z}^+| \leq |\mathbb{R}|$



one to one mapping

$\rightarrow$ $\therefore$ $|\mathbb{Z}^+| < |\mathbb{R}|$

$$|\mathcal{P}(\mathbb{Z}^+)| = 2^{|\mathbb{Z}^+|}$$

Cardinalities

$$\underbrace{\boxed{0 < 1 < 2 < 3 < 4 < \dots}}_{\text{finites}} < \underbrace{|\mathbb{Z}^+|}_{=\aleph_0} < |\mathbb{R}| = \aleph_1 = 2^{\aleph_0}$$

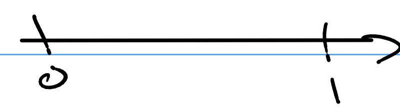
$0 < 1 < 2 < 3 < 4 < \dots < |\mathbb{Z}^+|$: countable

$|\mathbb{R}| = \aleph_1 = 2^{\aleph_0}$: uncountable

$$0 < 1 < 2 < 3 < \dots < \boxed{\aleph_0 < \aleph_1} < \aleph_2 < \dots$$

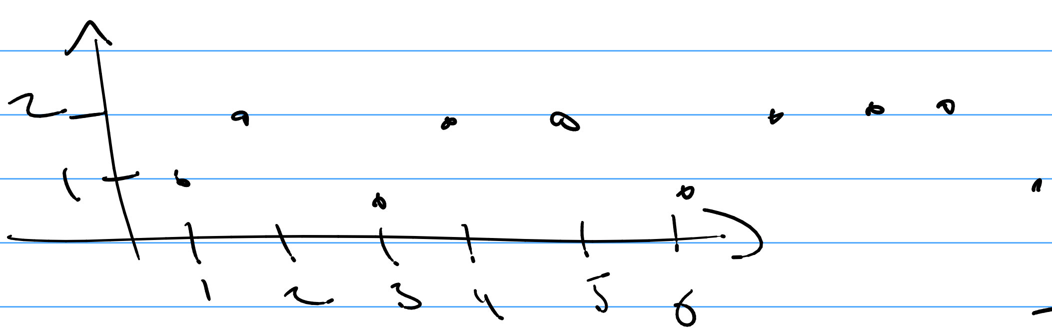
transfinites

$$\underset{\substack{\aleph_1 \\ \text{uncountable}}}{\mathbb{R}} = \underset{\substack{\aleph_0 \\ \text{countable}}}{\mathbb{Q}} \cup \underset{\substack{\aleph_1 \\ \text{uncountable}}}{\boxed{\text{irrationals}}}$$



→ Programming → Math instructions are countable

$0.121221222122221\dots$



$n = \bar{n} + \varepsilon$

$\boxed{\text{Linear Algebra}}$ Math 511 = toys + rules

Matrices → rectangular groups & numbers = , + , ·

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

size $m \times n$

a_{ij} are $\mathbb{R}$ → A is real valued

a_{ij} are $\{0,1\}$ (bits) → A is bit valued (called zero-one matrix)

ex

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad 2 \times 3$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad 3 \times 4$$

Ops | Real Values

$$A + B = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -3 & 1 \end{bmatrix}$$

$A \cdot B = \{c_{ij}\}$ $\quad c_{ij} = A$ i^{th} row (scalar prod) B j^{th} col

$$\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \boxed{1 \cdot 1 + 2 \cdot 0} \\ \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 3$

$$A^n = \underbrace{A \cdot A \cdot A \cdot \ldots A}_{n\text{-times}}$$

$$A^0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A is bit values (ops)

① $A \vee B = \{a_{ij} \vee b_{ij}\}$

② $A \wedge B = \{a_{ij} \wedge b_{ij}\}$

③ $A \odot B$

Ex

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} (1 \wedge 1) \vee (1 \wedge 1) & \cdots \\ (0 \wedge 1) \vee (1 \wedge 1) & \\ (1 \wedge 1) \vee (0 \wedge 1) & \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 2$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

4 $A^{\{n\}} = \underbrace{A \odot A \odot A \odot \cdots \odot A}_{n\text{-times}}$