Math 321

Q's 2.4 #38 use $\boxed{\text{telescoping}}$

$$\sum_{k=1}^{n} k^2 = ?$$

Know: $\sum_{k=1}^{n} 1 = n$, $\sum_{k=1}^{n} k = 1+2+\cdots+n = \frac{n(n+1)}{2}$

(37) $\boxed{(k)^2 - (k-1)^2} = (\cancel{k^2}) - (\cancel{k^2} - 2k + 1) = \boxed{2k-1}$

$$\boxed{\sum_{k=1}^{n} (k)^2 - (k-1)^2} = \sum_{k=1}^{n} \boxed{2k - 1}$$

$$(k)^3 - (k-1)^3 = \cancel{k^3} - (\cancel{k^3} - 3k^2 + 3k - 1)$$
$$= 3k^2 - 3k + 1$$

So $\boxed{\sum_{k=1}^{n} (k)^3 - (k-1)^3} = \sum_{k=1}^{n} 3\boxed{k^2} - 3k + 1$

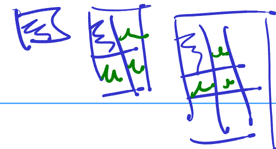
$$(\cancel{1^3} - 0^3) + (\cancel{2^3} - \cancel{1^3}) + (\cancel{3^3} - \cancel{2^3}) + (\cancel{4^3} - \cancel{3^3}) + \cdots + (n^3 - \cancel{(n-1)^3})$$

$n=1$ $\quad n=2$ $\quad n=3$ $\quad n=4$

$$n^3 - 0^3 = 3\sum_{k=1}^{n} k^2 - 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$n^3 = 3\boxed{\sum_{k=1}^{n} k^2} - 3\frac{n(n+1)}{2} + n$$

$$\sum_{k=1}^{n} \boxed{k^2} = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}\frac{n(n+1)}{2}(2n+1)$$



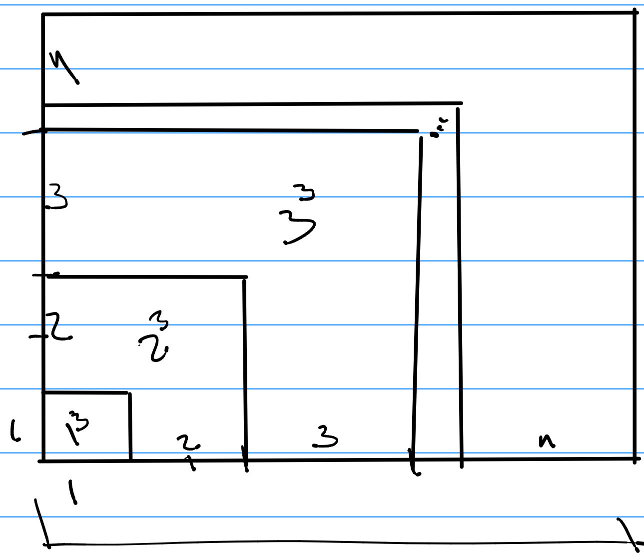
$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=1}^{5} k^3 =$$

$1^3+2^3+3^3+\dots+5^3$

$= \left(\frac{5(5+1)}{2}\right)^2$

5^{th} triangl.



$1+2+3+\dots+n = \frac{n(n+1)}{2}$

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26 AB defined

#11 $m\times n \quad k\times l$ $n=k$

A was $m\times n$

B was $k\times l$

BA defined if A was 3×4

$k\times l \quad m\times n$ $m=l$

B 4×3

AB 3×3

$3\times 4 \quad 4\times 3$

BA 4×4

$4\times 3 \quad 3\times 4$

2.6 #19

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Show that

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrices

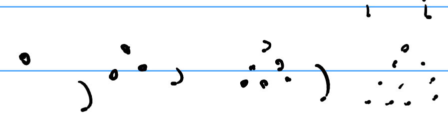
additive Identity $\quad 0 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & & 0 \end{bmatrix}$

mult. Identity $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2.5 #a

rooms $\quad$ 1, | 2, 3, | 4, 5, 6, | 7 8 9 10 | 11 12 13 14 15

	1	2	3	4		
bus 0	01,	02,	03,	04,	05,	06
bus 1	11,	12,	13,	14,	…	
bus 2	21,	22,	23,	24,	…	
bus 3	31,	32,	33,	34,	…	
⋮	⋮	⋮	⋮	⋮		



1 $\quad$ 3 $\quad$ 6 $\quad$ 10

Exam 2

11 probs @ 10pts

100 pts = 100%

2/1/22 Sets (4 probs)

① Representing Sets

roster/list → set-builder → Venn → roster/list

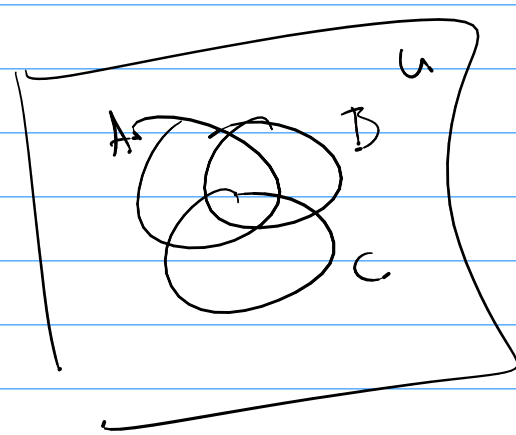
② $P(S) = ?$

$A_1 \times A_2 \times \cdots \times A_n = ?$

Ex $P(\{a,b\}) \times P(P(\emptyset))$

③ Membership Tables (+) Venn Diagrams

A	B	C	$A \cup (\overline{B \wedge C})$
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	



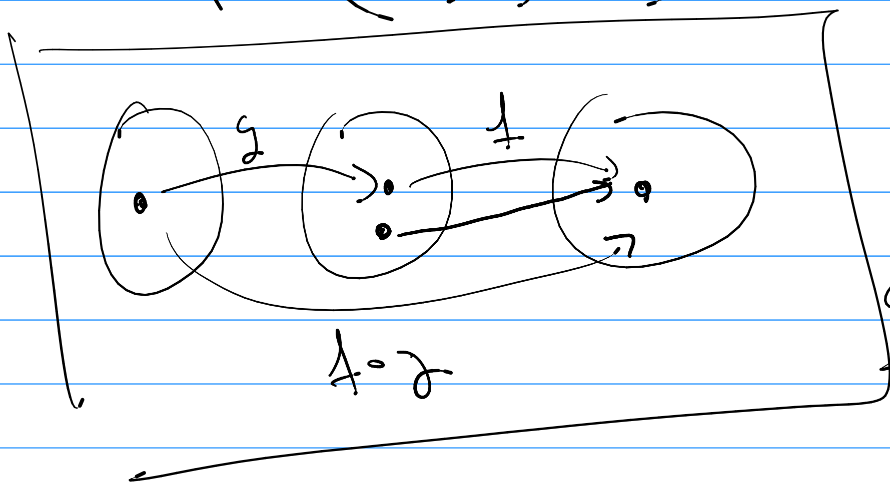
④ Show $S_1 = S_2$

a) by setbuilder notation and logic

or → b) show $S_1 \subseteq S_2 \wedge S_2 \subseteq S_1$

2.3 Functions (2 probs)

① Like p.154 (33, 34, or 35)



$f \circ g$

Counter example.

② State $f: A \to B$

a) Make an f that is / isn't onto and is / isn't one-to-one

b)

i. $f: \mathbb{Z}^+ \to \mathbb{Z}$

make an f that is onto and not one-to-one

2.4 Seq / Series (2 probs)

① seq → formula

formula → seq

② Sum

2.5 Cardinality (2 probs)

① $\mathbb{Q}$ is countable

② $\mathbb{R}$ is uncountable

[2.6] Matrices (1 prob)

Given Matrices .. do ops

$A+B$, AB, A^n, $A \wedge B$, $A \vee B$, $A \odot B$, $A^{[n]}$