

Math 321

Counting

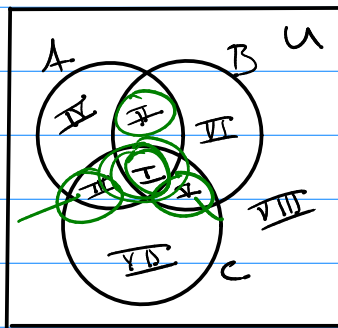
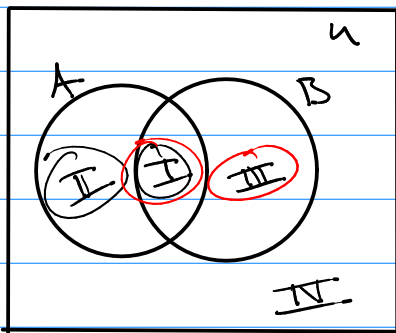
Product Rule: task that is using and actions
 \uparrow
 product

Sum Rule: task that is using or actions
 \uparrow
 sum

Overcounts

① Sum Rule Overcount (non-disjoint sets)

\rightarrow Inclusion/Exclusion Principle



2 Set Sum rule
 $|A \cup B| = |A| + |B|$
disjoint.

Count	A	B	A-B	B-C	C
I	1	1	1	0	0
II	1	0	1	0	0
III	0	1	0	1	0
IV	0	0	0	0	0

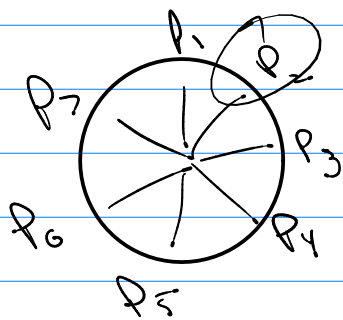
2 Set Incl/Exc Principle
 $|A \cup B| = |A| + |B| - |A \cap B|$

3 Set
 $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

② Product Rule over count \rightarrow Division Rule

\rightarrow task that by product rule has n ways to be done.

③ 7 people sit @ a round table.



$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7!$$

but

there is a way, w , that exactly d of the ways [correspond] to way w .

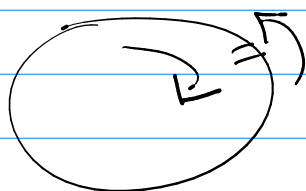
$$\text{total} = \frac{n}{d}$$

examples: "lead of the table" = 7 ways

\rightarrow also say 7 circular sym.

$$\text{total: } \frac{7!}{7} = \frac{7 \cdot 6 \cdot 5 \cdot \dots \cdot 1}{7} = 6!$$

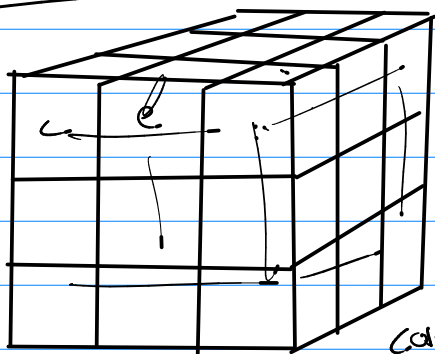
but if $P_2 - \text{me} \rightarrow P_1 = P_1 - \text{me} - P_2$ (2 rotational sym)



$$\begin{aligned} \frac{7!}{7 \cdot 2} &= \frac{6!}{2} = 6 \cdot 5 \cdot 4 \cdot 3 \\ &= 18 \cdot 20 \\ &= 360 \end{aligned}$$

Mohr's cube

(broken → put together)



$$| \text{corners} | = 8$$

$$| \text{edges} | = 12$$

$$\text{corners } (8 \cdot 3)(7 \cdot 3) \dots (1 \cdot 3) = 8! \cdot 3^8$$

$$\text{edges } (12 \cdot 2)(11 \cdot 2) \dots (1 \cdot 2) = 12! \cdot 2^{12}$$

$$\text{total } 8! \cdot 3^8 \cdot 12! \cdot 2^{12} \approx 519 \text{ quintillion}$$

but

Mohr's?

rotate = same

$$\frac{\text{do total}}{3+3+3+3} \approx \underline{\underline{43 \text{ quintillion}}}$$

28 numbers from 23 to 214 (inclusive)

how many are divisible by 3? $\frac{192}{3} =$

how many are divisible by 2? $\frac{192}{2} =$

how many are div. by 3 and 2? $\frac{192}{6} =$

" " " " " 3 or 2? =

$$\frac{192}{2} + \frac{192}{2} - \frac{192}{6} =$$

$$| \text{23 to 214} | = 214 - 23 + 1 = 192$$