Math 321

~~Q's~~ 6.1 #17 — 5 ASCII (128 ASCII)

how many contain '@' $\boxed{\text{at least once?}}$

$\boxed{\text{Notice!}}$ $\left|\text{all strings}\right| = \left|\begin{matrix}\text{no}\\ \text{'@'}\end{matrix}\right| + \boxed{\left|\begin{matrix}\text{exactly}\\ 1\ \text{'@'}\end{matrix}\right|} + \left|\begin{matrix}\text{exactly}\\ 2\ \text{'@'}\end{matrix}\right| + \dots + \left|\begin{matrix}\text{exactly}\\ 5\ \text{'@'}\end{matrix}\right|$

128^5 $\qquad 127^5$

5

$a_1\, a_2\, a_3\, @\, a_5$

127 127

task: get 1 '@' = 1 way

and put it in the string = 5 ways

and fill rest = 127^4

$$= \boxed{1 \cdot 5 \cdot 127^4}$$

$$\left|\text{at least one @}\right| = \left|\text{all strings}\right| - \left|\text{no '@'}\right|$$

$$= \boxed{128^5 - 127^5}$$

Now that we can count ... $|\text{Set}|$ = number

$\boxed{\text{Q}}$ is it enough to know $|S| = n$ to describe features of elements of S?

$\boxed{\text{Thm}}$ Pigeonhole/Dirichlet Drawer Principle $(k \geq 1)$

If $k+1$ or more objects are placed into k boxes

thus

at least one box has at least 2 objects.

$\boxed{\text{th}^{m}}$ (generalized)

If N objects are placed into K boxes
then at least one box has at least $\left\lceil \frac{N}{K} \right\rceil$ objects

ex 4 colors of socks

N-socks	$\lceil N/4 \rceil$
1	1
2	$1 = \lceil 2/4 \rceil$
3	$1 = \lceil 3/4 \rceil$
4	$1 = \lceil 4/4 \rceil$
5	$2 = \lceil 5/4 \rceil$
6	$2 = \lceil 6/4 \rceil$
7	
8	
9	
10	
11	

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4 colors -- $N = ?$ if I want 3 pairs of 1 color?

6 socks of 1 color

object box

$$\left\lceil \frac{N}{4} \right\rceil = 6$$

$$\left\lceil \frac{\overset{=21}{4 \cdot 5 + 1}}{4} \right\rceil = 6$$

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ex grades A, A-, B+, B, B-, C+, C, C-, D+, D, D-, F

How many students so at least 2 students have same grade? $\boxed{13}$

ex how many people so at least 2 have same first, last initials.

$26 \cdot 26 = 26^2$ boxes

$\boxed{26^2+1}$

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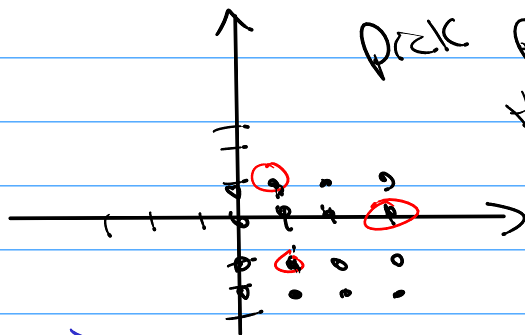
ex $\mathbb{Z} \times \mathbb{Z}$ plane



pick points so that a mid point is integer coords as well.

Midpoint $p_1 = (x_1, y_1)$

$p_2 = (x_2, y_2)$

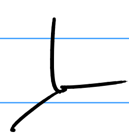
$M_{12} = \left(\boxed{\frac{x_1+x_2}{2}}, \boxed{\frac{y_1+y_2}{2}} \right)$

integer?

integer coord $\equiv$ x_1+x_2 is even, y_1+y_2 is even $\equiv$ x_1, x_2 same parity, y_1, y_2 same parity

Point: (even, even), (even, odd), (odd, even), (odd, odd) } 4 parity boxes → need $\boxed{5 \text{ points}}$

$\mathbb{Z}^3$ =



8 parity boxes → $\boxed{9 \text{ points}}$

$\mathbb{Z}^n$ 2^n parity boxes → $\boxed{2^n+1 \text{ points}}$

Applications of division rule.

ex 5 people: $|\text{arrangement}| = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 5!$

ex only give 1^{st}, 2^{nd}, 3^{rd} prize.

$$\frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}} = \boxed{5 \cdot 4 \cdot 3}$$

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}}$$