Math 321

Q's: $i_1 \le i_2 = i_1+1 \le i_3 = i_1+2 \le \dots \le i_n = i_1+(n-1)$

objects: integers → how many = n

boxes: remainders → how many = n

$\left\lceil \frac{n}{n} \right\rceil = 1$ → 1 integer for each remainder

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Socks: 5 red, 7 white, 9 blue

1 pair: objects = socks, boxes = colors $\left\lceil \frac{N}{3} \right\rceil = 2$ $\boxed{N=4}$

1 pair of blue: 5 red + 7 white + 2 blue = $\boxed{14}$

6.3

Permutations: $P(n,r) = \dfrac{n!}{(n-r)!}$ ← all arrange. of n

← divide out without order overcount

pick r with order from n

$(n-r)$ without order (overcount

ex: 15 people pick 5 to play basketball.

$$P(15,5) = \boxed{\frac{15!}{10!}}$$

Combinations

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

choose r without order from n objects

$n!$ ways arrange

choose r without order

$(n-r)$ without order

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ex 20 people = 13 guys, 7 girls

① choose 10 to play softball

$$C(20,10) = \frac{20!}{10!\,10!}$$

② choose 10 to play, but 1 must be a girl

at least

|exactly 1| + |exactly 2| + ... + |exactly 7|

$\binom{7}{2}\binom{13}{8}$ $\binom{7}{7}\binom{13}{3}$

$$\text{at least 1 girl} = \binom{7}{1}\binom{13}{9} + \binom{7}{2}\binom{13}{8} + \binom{7}{3}\binom{13}{7} + \dots + \binom{7}{7}\binom{13}{3}$$

$$= \frac{7!}{1!\,6!}\frac{13!}{9!\,4!} + \frac{7!}{2!\,5!}\frac{13!}{8!\,5!} + \dots + \frac{7!}{7!\,0!}\frac{13!}{3!\,10!}$$

or

|all teams| − |no girls| = |at least 1 girl|

$$= \binom{20}{10} - \binom{13}{10}$$

Count all at least 1 girl

example of a counting proof.
(or Combinatorial proof)

way #1 — way #2

$$\binom{20}{10} - \binom{13}{10} \stackrel{?}{=} \binom{7}{1}\binom{13}{9} + \binom{7}{2}\binom{13}{8} + \cdots + \binom{7}{7}\binom{13}{3}$$

$$\boxed{\binom{n+1}{k}} \stackrel{?}{=} \binom{n}{k} + 1\cdot\binom{n}{k-1}$$

n students + me