Math 321

Ch 6

→ Combinatorial Proofs / Counting proofs

$$|\text{task}|$$

these are "creative" proofs b/c you need figure out a task

$$|\text{using technique \#1}| = |\text{using technique \#2}|$$

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ex (creative aspect)

Know: $\boxed{1+2+3+\dots+n} = \boxed{\frac{n(n+1)}{2}}$

?? task ?? to do so that its cardinality

(1) can be counted by $1+2+3+\dots+$

or

(2) can be counted by $\frac{n\cdot(n+1)}{2} = \binom{n+1}{2}$

and

division rule.

Pascal's Identity $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

pore by Algebra $\frac{(n+1)!}{k!\,(n+1-k)!} \overset{?}{=} \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k-1)!\,(n-k+1)!}$

Combinatorial Proof: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

task: choose k people from $n+1$ for a committee.

n students + Mark.

tech #1 $\binom{n+1}{k}$

tech #2 all comm. of only students or have Mark

$\binom{n}{k} + 1 \cdot \binom{n}{k-1}$

pick Mark and rest are students

So $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

Show: $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = \boxed{n \cdot 2^{n-1}}$

task: of n people choose a committee and then its leader.

tech #1 $|\text{comm of } 1| + |\text{comm. of } 2| + |\text{comm of } 3| + \dots + |\text{com of } n|$

$1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n}$

tech #2 select leader and $P_2, P_3, P_4, \dots, P_n$

in out in out

$n \cdot 2 \cdot 2 \cdot \dots \cdot 2 = n2^{n-1}$

So $\binom{n}{1} + 2\binom{n}{2} + \dots + n\binom{n}{n} = n2^{n-1}$; $n = 1, 2, 3, \dots$

6.4 Binomial th^m

given $\square$'s and Δ's

<u>task:</u> choose n of them.

tech #1

choose $n - \square$'s or choose $(n-1) - \square$'s $\wedge$ $1 - \Delta$

or choose $(n-2)$ $\square$'s $\wedge$ $2 - \Delta$'s

$\vdots$

or choose $n - \Delta$'s

$$\frac{n!}{n!\,0!}\square^n + \frac{n!}{(n-1)!\,1!}\square^{n-1}\Delta^1 + \frac{n!}{(n-2)!\,2!}\square^{n-2}\Delta^2 + \cdots + \frac{n!}{0!\,n!}\Delta^n$$

tech #1

$$(\square + \Delta)^n = \underbrace{(\square+\Delta)\cdot(\square+\Delta)\cdot\ \ldots\ \cdot(\square+\Delta)}_{n-\text{times}}$$

$$\boxed{(\square+\Delta)^n = \frac{n!}{n!\,0!}\square^n + \frac{n!}{(n-1)!\,1!}\square^{n-1}\Delta^1 + \frac{n!}{(n-2)!\,2!}\square^{n-2}\Delta^2 + \cdots + \frac{n!}{0!\,n!}\Delta^n}$$

Binomial th^m

$$(\square + \Delta)^n = \sum_{k=0}^{n} \binom{n}{k} \square^{n-k} \Delta^k$$

VS $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

Pascal's triangle $\rightarrow$

$$(x+y)^5 = x^5 + \frac{5!}{4!1!}x^4y + \frac{5!}{3!2!}x^3y^2 + \frac{5!}{2!3!}x^2y^3 + \dots$$

Use binomial thm

$$(2x - 3x^{-1})^4$$

$$= (2x)^4 + \frac{4!}{3!1!}(2x)^3(-3x^{-1})^1 + \frac{4!}{2!2!}(2x)^2(-3x^{-1})^2$$

$$+ \frac{4!}{1!3!}(2x)^1(-3x^{-1})^3 + \frac{4!}{0!4!}(-3x^{-1})^4$$

$=$ Simplify!

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let $\square = 1$ $\Delta = 1$

$$(\square+\Delta)^n = \square^n + \frac{n!}{(n-1)!1!}\square^{n-1}\Delta + \dots + \Delta^n$$

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

$(\square+\Delta)^0$ $\quad \binom{0}{0}$

$(\square+\Delta)^1$ $\quad \binom{1}{0} \quad \binom{1}{1}$

$(\square+\Delta)^2$ $\quad \binom{2}{0} \quad \binom{2}{1} \quad + \quad \binom{2}{2}$

$(\square+\Delta)^3$ $\quad \binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

$(\square+\Delta)^4$ $\quad \binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$

1 → 1

1 1 → 2

1 2 1 → 4

1 3 3 1 → 8

1 4 6 4 1 → 16

1 5 10 10 5 1 → 32

1 1 2 3 5 8 13

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1