Math 321

Q5 (6.4 #19)

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

$$\frac{(n+1)!}{k!\,(n+1-k)!} \stackrel{?}{=} \frac{n!\,(n-k+1)}{k!\,\boxed{(n-k)!}} + \frac{k\,n!}{\boxed{k\,(k-1)!\,(n-k+1)!}}$$

$(n-k+1)$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \stackrel{?}{=} \underline{\text{Finish!}}$$

I know

$$\binom{a}{b} = \frac{a!}{b!\,(a-b)!}$$

$\boxed{q! = q\,(q-1)!}$

$(s+3)! = (s+3)(s+2)\,(s+1)!$

$(2z-1)! = (2z-1)(2z-2)!$

#25 $$\binom{2n}{n+1} + \binom{2n}{n} = \frac{1}{2}\binom{2n+2}{n+1}$$

$$\frac{(2n)!}{(n+1)!\,(2n-n-1)!} + \frac{(2n)!}{(n)!\,(2n-n)!} \stackrel{?}{=} \frac{1}{2}\,\frac{(2n+2)!}{(n+1)!\,(2n+2-n-1)!}$$

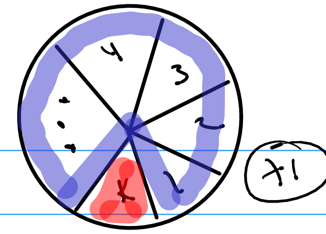
$$\frac{(2n)!}{(n+1)!\,(n-1)!} + \frac{(2n)!}{(n)!\,(n)!} \stackrel{?}{=} \frac{1}{2}\,\frac{(2n+2)!}{(n+1)!\,(n+1)!}$$

n

$(n+1)$

Kids, cookies, mouse

k kids



Basis: $k \mid a_0$

Inductive: $a_n = \left(\frac{k}{k-1}\right)a_{n-1} + 1$, $n = 1, 2, \dots, k$

$(k-1)^{k-1} \cdot \boxed{C} \mod k$

Soln: $a_0 = \begin{cases} (k-1)^{k} - (k-1) & \text{if } k \text{ is odd} \\ (k-1)^{k+1} - (k-1) & \text{if } k \text{ is even} \end{cases}$

rec: $a_n = \left(\frac{k}{k-1}\right)(a_{n-1}) + 1 \quad n = 1, 2, \dots, k$

closed: $a_n = \begin{cases} k^n (k-1)^{k-n} - (k-1) & k \text{ odd } \& \\ k^n (k-1)^{k-n+1} - (k-1) & k \text{ even} \end{cases}$

Solve Recursive Relations

Solve: $a_n =$ expression with $a_{n-1}, a_{n-2}, \dots, a_{n-k}$, stuff

→ $a_n =$ function of n

How? (1) $\boxed{\text{Guess}}$ → restrict rec. relation to classes & types.

(2) Iteration

restrict. Linear homogeneous rec. relation of degree k with constant coeff.

8.2 $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

$c_i \in \mathbb{R} \qquad c_k \neq 0$

ex $a_n = (1) a_{n-1} + (1) a_{n-2}$

ex $a_n = a_{n-2} - 3 a_{n-7}$

ex $a_n = a_{n-4} + \pi a_{n-5} + a_{n-11}$

$$\boxed{\text{Soln: } \quad a_n = r^n \qquad r \in \mathbb{R}}$$

Solved

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

$r^n = c_1 \frac{r^n}{r} + c_2 \frac{r^n}{r^2} + \dots + c_k \frac{r^n}{r^k}$

$1 = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots + \frac{c_k}{r^k}$

$r^k = c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_k$

$$\boxed{r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k = 0}$$ characteristic eqn

Solve:

$r_1, r_2, \dots, r_k$

$a_n = r_i^n$ are solutions

$0, 1, 1, 2, 3, 5, 8, \ldots$

ex $\quad a_n = a_{n-1} + a_{n-2} \qquad a_0 = 0, \; a_1 = 1$

Sol $\quad a_n = r^n$

$$r^n = r^{n-1} + r^{n-2} \qquad 1 = \frac{1}{r} + \frac{1}{r^2}$$

$$\cancel{r^n} = \frac{\cancel{r^n}}{r} + \frac{\cancel{r^n}}{r^2} \qquad r^2 = r + 1$$

charc. eqn $\boxed{r^2 - r - 1 = 0}$

Note: all charc. eqn's $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \ldots - c_k = 0$

$$r = \frac{1 \pm \sqrt{1+4}}{2}$$

$$r_1 = \frac{1+\sqrt{5}}{2} \qquad r_2 = \frac{1-\sqrt{5}}{2}$$

$$a_n = c_1\left(\frac{1+\sqrt{5}}{2}\right)^n + c_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\begin{cases} 0 = c_1 + c_2 \rightarrow c_2 = -c_1 \\ 1 = c_1\left(\frac{1+\sqrt{5}}{2}\right) - c_1\left(\frac{1-\sqrt{5}}{2}\right) \end{cases}$$

$$2 = \cancel{c_1} + c_1\sqrt{5} \cancel{- c_1} + c_1\sqrt{5}$$

$$\frac{1}{\sqrt{5}} = c_1 \qquad c_2 = -\frac{1}{\sqrt{5}}$$

$$\boxed{a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$