Math 321

Q's

6.4 #11 $\left(x^2 + (-x^{-1})\right)^{100}$

$$= (x^2)^{100} + 100(x^2)^{99}(-x^{-1})^1 + \frac{100!}{98!\,2!}(x^2)^{98}(-x^{-1})^2 + \dots$$

$$+ \boxed{\frac{100!}{(100-j)!\,j!}(x^2)^{100-j}(-x^{-1})^j} + \dots + (-x^{-1})^{100}$$

$$= x^{200} - 100x^{197} + \frac{100!}{98!\,2!}x^{194} - \dots + x^{-100}$$

$$\boxed{(-1)^j \frac{100!}{(100-j)!\,j!} x^{200-3j}}$$

x^k

$200 - 3j = k$

$\frac{200-k}{3} = j$

x^{-5} $\quad j = \frac{200-(-5)}{3} = \frac{205}{3} = 68$

if $3 \mid 200-k$

use $j = \boxed{\frac{200-k}{3}}$ $\quad (-1)^j \frac{100!}{(100-j)!\,j!} x^k$

if $3 \nmid 200-k$ coef $= 0$

8.2 rec. relation: $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

Basis b/c degree k, need k initial values

$a_0, a_1, a_2, \dots, a_{k-1}$

Solve   Sol'n's $a_n = r^n$ (find all r's)

plug it:

get characteristic eq'n $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k = 0$

→ has $r_1, r_2, \dots, r_k$ solutions.

so $a_n = (r_1)^n, \; a_n = (r_2)^n, \; \dots, \; a_n = (r_k)^n$

→ how to get one formula?

$$\boxed{a_n = ?}$$

1st $r_1, r_2, \dots, r_k$ are all uniq.

ans $a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n + \dots + \alpha_k (r_k)^n$

make a linear combination

2nd $r_1, r_2, \dots, r_k$ are not all uniq.

ex $r_1 = 2, \; r_2 = 2, \; r_3 = 2, \; r_4 = (-1)$

$a_n = (2)^n \quad a_n = (2)^n \quad a_n = (2)^n \quad a_n = (-1)^n$

$a_n = \alpha_1 (2)^n + \alpha_2 \cdot n (2)^n + \alpha_3 n^2 (2)^n + \alpha_4 (-1)^n$

$$\rightarrow \boxed{a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2)(2)^n + \alpha_4 (-1)^n}$$

polynomial combination.

ex $a_n = 4a_{n-1} + 3a_{n-2} - 18a_{n-3}$ degree $= 3$

Sol'n $a_n = r^n$

char. eqn's $r^3 - 4r^2 - 3r + 18 = 0$

$(r+2)(r^2 - 6r + 9) = 0$

$(r+2)(r-3)(r-3) = 0$

$r = -2 \quad r = 3 \quad r = 3$

$$\boxed{a_n = \alpha_1(-2)^n + (\alpha_2 + \alpha_3 n)(3)^n}$$

Basis: (Initial Values) $a_0 = 0 \quad a_1 = 1 \quad a_2 = 1$

$$\begin{cases} 0 = \alpha_1 + \alpha_2 \\ 1 = -2\alpha_1 + 3\alpha_2 + 3\alpha_3 \\ 1 = 4\alpha_1 + 9\alpha_2 + 18\alpha_3 \end{cases}$$

$\Rightarrow \alpha_1 = ? \quad \alpha_2 = ? \quad \alpha_3 = ?$

Non-Homogeneous?

$$a_n = \underbrace{c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}}_{\text{associated homogeneous relation}} + F(n)$$

Solution: ① $a_n^{(h)}$ Solve the assoc. homogeneous relation

② "guess" or "find" some particular solution to

the $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n)$

$a_n^{(p)}$

all sol'n's $\boxed{a_n = a_n^{(p)} + a_n^{(h)}}$
