Math 321

8.1 #1

$$\boxed{\begin{aligned} H_n &= 2H_{n-1}+1 \\ H_1 &= 1 \end{aligned}} \to \underline{\text{seq}}: \; 1, 3, 7, 15, \ldots, 2H_{k-1}+1$$

$H_1, H_2, H_3, H_4, \ldots \quad H_k$

Formula: $\boxed{H_n = 2^n - 1}$

Inductive Proof

Basis: 1^{st} $H_1 = 1 \overset{?}{=} 2^1 - 1$ true.

Inductive: assume k^{th} formula works up to the k^{th} $H_k = 2^k - 1$

Show $k+1^{st}$ "show formula works for next number" H_{k+1}

open form: $H_{k+1} = 2(H_k) + 1 = 2(2^k - 1) + 1$

IH says..

$H_{k+1} = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

so formula works

true

8.1 #4

\$n \$1 or \$2 or \$5 or \$10 or \$20 or \$50 or \$100

$a_n = 1\cdot a_{n-1} + 1\cdot a_{n-2} + 2\cdot a_{n-5} + 2\cdot a_{n-10} + 1\cdot a_{n-20} + 1\cdot a_{n-50} + 1\cdot a_{n-100}$

$a_0 = 1$

$a_1 = 1\cdot a_0 = 1$

$a_2 = 1\cdot a_1 + 1\cdot a_0 = 2$

$a_3 = a_2 + a_1 = 3$

$\vdots$

Exam 4

11 probs @ 10 pts

100 pts = 100%

6.1 Sum, Product, Inclusion/Exclusion, Division Rules

3 prob

① basic use sum/product rules

②/③ overcount type problems.

ex how many numbers between 12 ... 1024 are div. by 2 or 3?

ex how many ways to put 11 people around a round table? (if same people on either side a person is same arrangement)

6.2 Pigeonhole Principle

1 problem

① a use of generalized version.

6.3 $P(n,r) = \frac{n!}{(n-r)!}$, $C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

2 probs

6.4 Binomial thm

2 probs

① use $(a+b)^n = \ldots$

$$((2x)-(\tfrac{3}{x}))^{20} = \boxed{\quad ? \quad}$$

② Combinatorial Proof

Ch 8

8.1 / 8.2

3 probs

① given problem find a rec. relation model.
(maybe with the basis)

②③ Solve rec. relation.

quadratic: $a_n = a_{n-1} + 2a_{n-2}$

Sol: $a_n = r^n$

$r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r = 2 \qquad r = -1 \qquad a_n = (2)^n \qquad a_n = (-1)^n$

$$\boxed{a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n}$$

given initial values $a_0 = 1 \quad a_1 = 1$

$$\begin{cases} 1 = \alpha_1 + \alpha_2 \\ 1 = 2\alpha_1 - \alpha_2 \end{cases}$$

$2 = 3\alpha_1 \rightarrow \alpha_1 = 2/3 \qquad \alpha_2 = 1/3$

$$\boxed{a_n = \tfrac{2}{3}(2)^n + \tfrac{1}{3}(-1)^n}$$