

Online Math 321

Final Exam Review

21 problems @ 10 pts each
190 pts = 100%

From Midterm (3 problem types)

① English sentence \rightarrow compound proposition \leftrightarrow truth table

ex: "bkh..." P : "I am picky"
 Q : "I run quickly"

$$(\neg P \rightarrow Q) \vee \neg Q$$

P	Q	$(\neg P \rightarrow Q) \vee \neg Q$
T	T	
T	F	
F	T	
F	F	

etc

② Prove: $\square \leftrightarrow \Delta$

Case 1 $\square \rightarrow \Delta$

Case 2 $\Delta \rightarrow \square$

③ Prove \mathbb{Q} are countable
or

Prove \mathbb{R} are uncountable

Rest are Ch 4, 5, 6, 8

4.1 Divisibility (2 probs)

(1) Proof using alb type problem

$$\text{ex } \boxed{a|b \wedge b|c} \rightarrow \boxed{a|c}$$

PF use $\Delta | \Delta$ is $B = k = 1$, $k \in \mathbb{Z}$

$$a \cdot k_1 = b \wedge b \cdot k_2 = c$$

$$\text{so } a \cdot k_1 \cdot k_2 = c \rightarrow a(k_1 k_2) = c \rightarrow a|c$$

□

(2) Div, Mod, $a \equiv b \pmod{n}$ or $a \equiv b \pmod{m}$

4.2 ()_b base b numbers (0 probs)

4.3 Primes (3 probs)

(1) Prove there are ∞ primes.

(2) gcd, lcm using prime factors

(3) gcd using Euclid's Alg.

4.6 RSA Encryption

$$E(p) = p^e \pmod{n}$$

$$E^{-1}(c) = c^d \pmod{n}$$

$$n = p \cdot q$$

$$d \text{ is } e\text{'s inv. mod } (p-1)(q-1)$$

give you $e, n \rightarrow$ you find p, q, d .

5.1 (5.2) Induction (2 probs)

(1) Equality proof using weak induction

(ex) $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

pf: Basis: $1 \stackrel{?}{=} \frac{1(1+1)}{2} \quad \underline{\underline{\text{true}}}$

Inductive assume I.H. $1 + 2 + \dots + k = \frac{k(k+1)}{2}$

now show $1 + 2 + \dots + (k+1) = \frac{(k+1)(k+2)}{2}$

Finish

(2) Pex: $n \geq 2$, n is prime or a product of primes
(strong induction)

6.1 Counting: Sum, Product, Incl/Exc, Division

(2 probs)

(1) Application without overcounts

(2) Incl/Exc or Division rule

6.2 Pigeonhole Principle (2 probs)

(1) application of generalized version

(ex) how many people to have 6 with same Feb initials?

② #10 p. 405

6.3 $P(n,r) = \frac{n!}{(n-r)!}$ $C(n,r) = \frac{n!}{r!(n-r)!}$

2 probs

① Word problem using $P(n,r)$, $C(n,r)$

② Word problem using $C(n,r)$ ✗

ex 5 blue blocks, 10 red blocks

choose 4 blocks if at least 3 must be red?

6.4 Binomial th^m 2 probs

① Combinatorial proof

② application $(a+bx)^n$

ex $(x + \frac{2}{x})^7 = ?$

ex $(3x - \frac{1}{x^3})^{100}$, the 41st term?

6.1/6.2

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$$

2 probs

$a_n =$ rec. relation.

① Model using rec. relation.

ex $H_n = ?$ for tower of Hanoi.

Rec $\left. \begin{array}{c} | \\ | \\ | \\ \vdots \\ | \end{array} \right\} n\text{-disks}$

Rec. relation $H_n = H_{n-1} + 1 + H_{n-1}$
 $\uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
move $n-1$ move move $n-1$
bottom between on top of bottom

Basis $H_1 = 1$

② Solve a $a_n = C_1 a_{n-1} + \dots + C_k a_{n-k}$

ex $a_n = a_{n-1} + 2a_{n-2}$

\rightarrow Soln $a_n = r^n$

$r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r=2 \quad r=-1 \quad (2)^n, (-1)^n$

$a_n = d_1(2)^n + d_2(-1)^n$ (if no basis elements given)

find d_i if given basis values.
