Math 511

row echelon form / Gaussian Elimination

System of linear eqn's

augmented matrix $[A \mid b] \to \left[\begin{array}{cccc|c} x & x & \cdots & & x \\ 0 & 0 & x & & x \\ 0 & 0 & 0 & x & x \end{array}\right]$

numbers

solve using back substitution

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vs $[A \mid b] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \to \begin{array}{l} x=1 \\ y=2 \\ z=3 \end{array}$

reduced row echelon form.

ex

$$\begin{array}{c} \begin{array}{ccccc} x & \boxed{y} & z & \boxed{s} & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

let $y = c_1$ $\quad s = c_2$

$z = 2 - c_2$

$x = 5 - 2c_1 - 4c_2$

$$\boxed{\begin{aligned} x &= 5 - 2c_1 - 4c_2 \\ y &= c_1 \\ z &= 2 - c_2 \\ s &= c_2 \end{aligned}}$$

augmented matrix ← gauss-jordan elimination

$$[A \mid b] \xrightarrow[\text{row ops}]{\text{by using elem.}} [\text{reduced row echelon form} \mid \ ]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ -1 & 2 & 1 & 1 \\ 3 & 1 & 0 & 5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -4 & -5 \\ 0 & 5 & 9 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 5 & 9 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 3/16 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 3/16 \end{array}\right]$$

gaussian to get row echelon

Continue ← Jordan part & Gauss-Jordan

Homogeneous Systems → all $\boxed{b_i = 0}$

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 + \dots + a_{2n}x_n = 0 \\ \vdots \\ a_{n1}x_1 + \dots + a_{nn}x_n = 0 \end{cases}$$

ex: $\begin{cases} x + y = 0 \\ x - y = 0 \end{cases}$

are all consistent (b/c at least $x_1 = 0, x_2 = 0, \dots, x_n = 0$ is a soln)

Note

$x^2 - x - 6 = 0$ $\quad a \cdot b = 0$

$(x-3)(x+2) = 0$

$\to x-3=0 \quad x+2=0$

$x=3 \quad x=-2$

Augmented Matrices $\to$

$x + y = 1$

$2x - y = 3$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & 3 \end{array}\right]$$

objects: Matrices / Vectors

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Notation Matrix: capital letters.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$ size $m \times n$

$A = [a_{ij}]$

$Q = [q_{ij}]$

Vectors: row vectors, col. vectors

$$[x_1\ x_2\ \dots\ x_n] \qquad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

$1 \times n \qquad m \times 1$

Notation: col. vector lower case bold font $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}$

row vector $\vec{v}$

So $A = [a_{ij}] = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = [\mathbf{a}_1\ \mathbf{a}_2\ \cdots\ \mathbf{a}_n]$

$= \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix}$

Rules (operations)

① (Sameness) $A = B$ same size (both are $m \times n$)

$a_{ij} = b_{ij}$ for each i, j

② $A + B = [a_{ij} + b_{ij}]$

both $m \times n$

③ αA, α is a scalar (scalar multiplication)

$\alpha A = [\alpha a_{ij}]$

④ $A - B = A + (-1)B = [a_{ij} - b_{ij}]$

⑤ Mutiplication?

Matrix multiply Matrix?

$$a_1x_1 + a_2x_2 = b$$

Def $\vec{a_1} = [a_1 \; a_2]$

Scalar product $\vec{a_1} \cdot x = b$

$$[a_1 \; a_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b$$

$$\to a_1x_1 + a_2x_2 = b$$