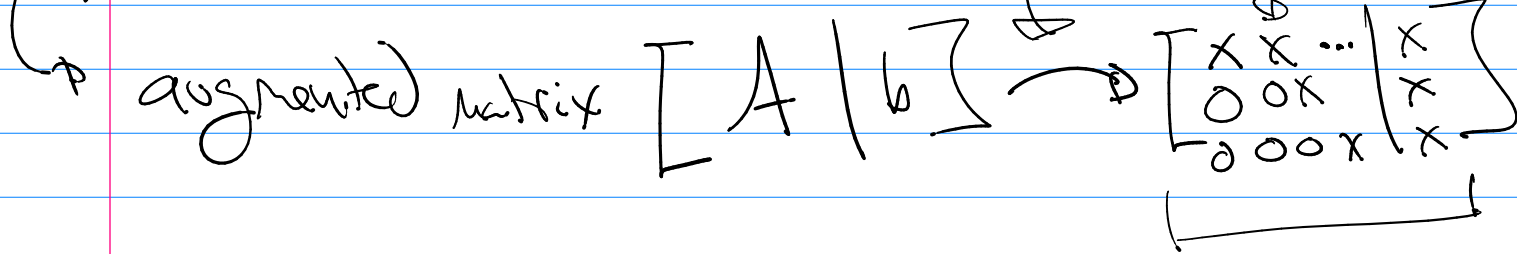


Math 511

row echelon form / Gaussian Elimination

System of linear eq's



Solve using
back substitution



reduced row echelon form.

ex $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

Let $y = c_1$ $s = c_2$

$z = 2 - c_2$

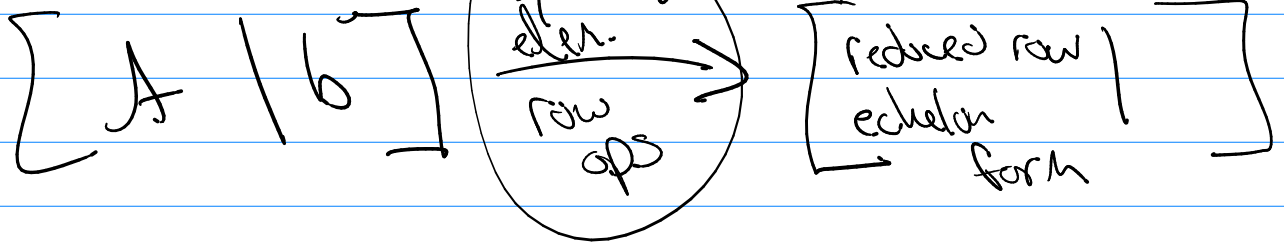
$x = 5 - 2c_1 - 4c_2$

$y = c_1$

$z = 2 - c_2$

$s = c_2$

augmented matrix \rightarrow gauss-jordan elimination



$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ -1 & 2 & 1 & 1 \\ 3 & 1 & 0 & 5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -4 & -5 \\ 0 & 5 & 9 & 7 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 5 & 9 & 7 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 3/16 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 3/16 \end{array} \right]$$

gaussian to get row echelon

Complete Jordan part & Gauss-Jordan

homogeneous systems

all $b_i = 0$

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 + \dots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n = 0 \end{cases}$$

ex $\begin{cases} x + y = 0 \\ x - y = 0 \end{cases}$

are all consistent (b/c at least $x_1=0, x_2=0, \dots, x_n=0$ is a soln)

Note

$$x^2 - x - 6 = 0$$

$$a \cdot b = 0$$

$$(x-3)(x+2) = 0$$

$$\begin{cases} x-3=0 & x+2=0 \end{cases}$$

$$x=3$$

$$x=-2$$

Augmented Matrices

$$\rightarrow \begin{cases} x+y=1 \\ 2x-y=3 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & 3 \end{array} \right]$$

directly Matrices / Vectors

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Note

Matrix : capital letters

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \quad \text{size } m \times n$$

$$A = [a_{ij}]$$

$$Q = [q_{ij}]$$

Vectors:

row vectors,

col. vectors

$$[x_1 \ x_2 \ \dots \ x_n]$$

$1 \times n$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$n \times 1$

Notation:

col. vector

lower case bold font

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

row vector

\vec{x}

So

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} = [a_{11} \ a_{12} \ \dots \ a_{1n}]$$

$$= \begin{bmatrix} \vec{a}_{1j} \\ \vdots \\ \vec{a}_{mj} \end{bmatrix}$$

Rules (operations)

① (Sameness) $A = B$ same size (both are $m \times n$)

$a_{ij} = b_{ij}$ for each i, j

② $A + B = [a_{ij} + b_{ij}]$

both $m \times n$

③ αA , α a scalar (scalar multiplication)

$$\alpha A = [\alpha a_{ij}]$$

$$\textcircled{4} \quad A - B = A + (-1)B = [a_{ij} - b_{ij}]$$

$\textcircled{5}$ Multiplication?

Matrix multiply Matrix?

$$a_1x_1 + a_2x_2 = b$$

Def

$$\vec{a}_1 = [a_1 \ a_2]$$

Scalar Product

$$\vec{a}_1 \cdot \vec{x} = b$$

$$[a_1 \ a_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b$$

$$\rightarrow a_1x_1 + a_2x_2 = b$$