Math 511

Q's / $\boxed{O}$?

Identities | Inverses

given an operation and object can you make the identity?

with an operation what object does nothing?

ex College Algebra

additive identity

① addition: $+$ $\qquad X + \boxed{0} = X$

Inverse? $\qquad X + (5 + (-5))$

additive inverse of 5.

$x + 0$

x

② Calculus $(f \circ g)(x) = f(g(x))$

$f(g(x)) = f(x)$

$g(x) = x$ Identity of function composition

Matrix Algebra

Identity?

① $A + B$ $\qquad A + [\,?\,] = A$

$= [a_{ij} + b_{ij}]$

want $A + [0\text{'s}] = [a_{ij} + 0] = [a_{ij}]$

define all zero matrix $O = [0]$

& additive identity

$$A + O = O + A = A$$

additive inverse of A?

$$A + [?] = O$$

$$[a_{ij} + \underline{\underline{-a_{ij}}}] = [0]$$

so A's add inverse is $(-1)A$.

Matrix Multiplication identity (inverse?)

Identity

$$\underset{m\times n}{A} \cdot \underset{n\times n}{[?]} = \underset{m\times n}{A}$$

$$\underset{m\times m}{[?]} \underset{m\times n}{A} = \underset{m\times n}{A}$$

def

$$\underset{n\times n}{I_n} = \begin{bmatrix} 1 & \ddots & 0 \\ 0 & & 1 \end{bmatrix} = [e_1 \; e_2 \; \cdots \; e_n]$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ex
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Matrix Multiplicative Inverse?

$$A\,(A\text{'s inv}) = I$$

$$(A\text{'s inv})\,(A) = I$$

When does A have an inverse?

1. A is $n \times n$
2. ? need more work

Def call A's matrix mult. inverse to be A^{-1}

and $A \cdot A^{-1} = A^{-1} \cdot A = I$

if A^{-1} exists we call A invertable or non-singular

otherwise A is called non-invertable or singular

Idea: College Algebra Zero has no mult. inv.

b/c $x \cdot 0 = 0$

0

ex
$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

this is singular

$$0 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thm: if B^{-1}, A^{-1} exist then

$$(AB)^{-1} = B^{-1}A^{-1}$$

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pf

$$(AB)^{-1}(AB) = B^{-1}\underbrace{A^{-1}A}_{I}B = I$$

1.6 system of eqn $\rightarrow$ aug. matrix gaussian $\rightarrow$ elimination solve by back sub.

elem. row ops

$Ax = b$

$EAx = Eb$

Goal: solve $Ax = b$

if A^{-1} existed and we knew it

$$A^{-1}Ax = A^{-1}b$$
$$Ix = A^{-1}b$$
$$x = A^{-1}b$$

elem. row ops $\longleftrightarrow$ elem matrix

① row swap $\Longleftrightarrow$ E_{type1} take I swap row i and row j

ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $E_{type1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

swap row 1, row 2

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

Does E_{type1} have an inverse?

Yes $E_{type1}^{-1} = E_{type1}$

② $c \cdot row_i = new\ row_i$ $\Longleftrightarrow$ E_{type2} is I but replace 1 on row i with c

ex: $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

so $\vec{E}_{type2}$ is E_{type2} <u>but</u> c is $1/c$

<u>ex</u> $E_{type2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\vec{E}_{type2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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(3) $row_i = row_i + c\,row_j \iff E_{type3}$ is I but put c in a_{ij} position

ex $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

row 3 = row 3 + 2 row 2

$\vec{E}_{type3}$ is E_{type3} <u>but</u> $-c$ is the c spot

$\to E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$
