Math 511

Q3/ strictly triangular $\quad \det(A) = \dfrac{(1)(-5)(-58/5)}{1\cdot 1\cdot 1\cdot 1} = \boxed{58}$

Ex $\det(A)$ by elimination $\quad \det(A) = \dfrac{\det(U)}{\det(E_1)\det(E_2)\cdots\det(E_k)}$

$$A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & 2 \\ 5 & -1 & -4 \end{bmatrix} \xrightarrow{r_1 = r_1 + (-1)r_2} \begin{bmatrix} 1 & 2 & -2 \\ 3 & 1 & 2 \\ 5 & -1 & -4 \end{bmatrix}$$

$$\xrightarrow[r_3 = r_3 + (-5)r_1]{r_2 = r_2 + (-3)r_1} \begin{bmatrix} 1 & 2 & -2 \\ 0 & -5 & 8 \\ 0 & -11 & 6 \end{bmatrix} \xrightarrow[r_3 + \left(\frac{-11}{5}\right) r_2]{} \begin{bmatrix} 1 & 2 & -2 \\ 0 & -5 & 8 \\ 0 & 0 & \frac{-58}{5} \end{bmatrix}$$

Ex $\det(A)$ by cofactors

row1 expansion

$$A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & 2 \\ 5 & -1 & -4 \end{bmatrix} = 4\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix} - 3\begin{vmatrix} 3 & 2 \\ 5 & -4 \end{vmatrix} + \cancel{0\begin{vmatrix} 3 & 1 \\ 5 & -1 \end{vmatrix}}$$

$$= 4(-4+2) - 3(-12-10)$$

$$= -8 + 66 = \boxed{58}$$

$$\det\begin{pmatrix} -12 & -1 & 4 \\ 6 & 2 & 1 \\ 3 & 1 & 1 \end{pmatrix} \qquad \det(A) = \frac{3\cdot 3\cdot(-1)}{(-1)(1)(1)(-1)} = -9$$

$$\begin{bmatrix} -12 & -1 & 4 \\ 6 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \xrightarrow[r_1, r_3]{\text{swap}} \begin{bmatrix} 3 & 1 & 1 \\ 6 & 2 & 1 \\ -12 & -1 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 3 & 8 \end{bmatrix}$$

$$u = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & 8 \\ 0 & 0 & -7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A = u$$

$$\underbrace{(E_k \cdots E_2 E_1)}_{= A^{-1}} A = I$$

$\boxed{\text{Exam 1}}$ 11 probs @ 10 pts each

100 pts = 100%

1.1 Systems of Eq's (solve by sub. or elimination)

1 prob $\rightarrow$ $\boxed{\text{Not}}$ matrices

1.2 System of Eq's $\boxed{\text{solve by aug. matrices}}$

gauss

and/or gauss jordan

2 probs

① (same as)

② word problem

1.3/1.4 Matrix Ops (Arithmetic/Algebra)

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}^T \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}^3 - 4\begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix} + \left(\underset{1\times2}{\begin{bmatrix} 1 & 2 \end{bmatrix}}\underset{2\times1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}\right)\left(\underset{2\times1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}\underset{1\times2}{\begin{bmatrix} 1 & 2 \end{bmatrix}}\right)$$

(2 probs)

1.5 Elem. Matrices (3 probs)

1. LU factorization

2. Find A^{-1}

3. 1.5 #15, #16 & homework (Variation)

Hint: Know thm 1.3.1, 1.5.2

1.3 / 1.4 / 1.5 (1 prob)

Matrix arith / algebra / Inverses ..

ex) Solve for X

all given matrices

$$AX + B = CX - 3D$$

$$(A-C)X = -3D - B$$

$$X = \boxed{(A-C)^{-1}(-3D-B)}$$

actual number matrices $\rightarrow \left[\, A - C \mid -3D - B \,\right]$

$$\rightarrow \left[\, I \mid \text{ans} \,\right]$$

no partitioned matrix probs

2.1/2.2 (2 probs)

1. 2. } find $\det(A)$ by elimination and/or cofactors

may or may not have numbers in it.

ex (at start of class)

ex $\det\begin{pmatrix} 1 & a & b \\ 2 & 3 & 1 \\ 3 & 1 & x \end{pmatrix}$ 4

plus understand what $\det(A) = 0$ or $\det(A) \neq 0$ means