Math 511

Q's 3.4 #7 $S = \left\{ \begin{bmatrix} a+b \\ a-b+2c \\ b \\ c \end{bmatrix} \,\middle|\, a,b,c \in \mathbb{R} \right\}$

$\boxed{\text{Note:}}$ ex $N(A) \to [A \mid 0]$ solve: $x = \begin{bmatrix} \alpha+\beta \\ \alpha-\beta+2\gamma \\ \beta \\ \gamma \end{bmatrix}$

all x such that $Ax = 0$

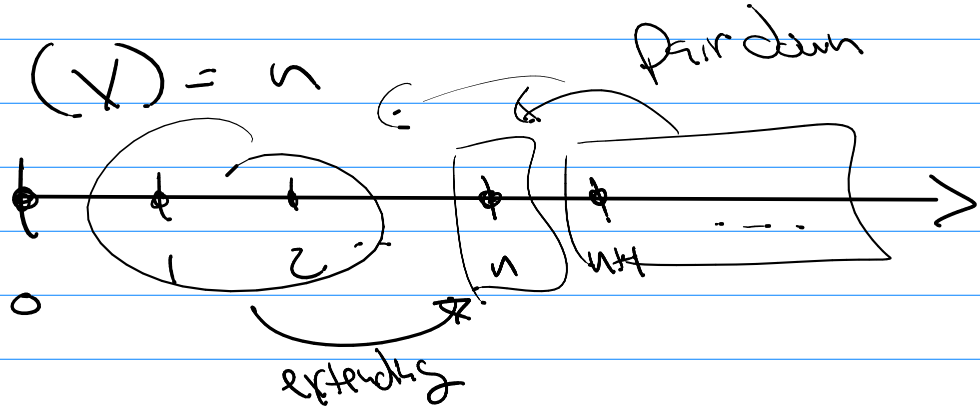
$$x = \begin{bmatrix} 1\cdot\alpha + 1\cdot\beta + 0\gamma \\ 1\alpha + (-1)\beta + 2\cdot\gamma \\ 0\cdot\alpha + 1\beta + 0\gamma \\ 0\alpha + 0\beta + 1\gamma \end{bmatrix} = \alpha\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + \beta\begin{bmatrix}1\\-1\\1\\0\end{bmatrix} + \gamma\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$$

So S is the Span $\left(\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\\1\end{bmatrix} \right)$

$\dim(S) = 3$

$\dim(\{0\}) = 0$

$\dim(V) = n$



(add linearly ind. vectors)

$\boxed{Th^n}$ $\dim(V) = n > 0$

(1) $\boxed{\text{any}}$ set of n linearly ind. vectors will span V.

(2) any n-vectors that span V will be linearly ind.

$\boxed{So}$ these n-vectors form a basis of V.

$\boxed{Th^n}$ $\dim(V) = n > 0$

(1) no set of less than n will span V

(2) any subset of fewer than n vectors can be $\boxed{\text{extended}}$ (add lin. ind. vectors) to form a basis

(3) any spanning set of more than n-vectors can be $\boxed{\text{paired down}}$ (remove dep. vectors). to get a basis

ex $\mathbb{R}^3$ $\left\{ \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \cancel{\begin{bmatrix}1\\2\\0\end{bmatrix}}, \cancel{\begin{bmatrix}0\\4\\0\end{bmatrix}}, \cancel{\begin{bmatrix}3\\3\\0\end{bmatrix}} \right\}$

$v_1 + v_2$ $\quad 4v_2$ $\quad 3v_1$

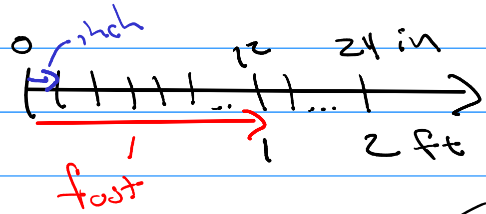
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 1 & 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathbb{R}^3$ $\left\{ \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix} \right\}$

[SB] $\dim(V) = n$ ← β uniq

but the basis $v_1, v_2, \ldots, v_n$ are [not] uniq.

Ex: 1D



coordinate

unit = basis

$[12]_{in}$ same as $[1]$ feet

$[1]_{in}$ same as $[2.54]_{cm}$

$(12)[12]_{ft} = [144]_{in}$ ft to in conversion

$[12]_{ft} = (\tfrac{1}{12})[144]_{in}$ in to ft conversion

$C[\quad]_{B_1} = [\quad]_{B_2} \to [\quad]_{B_1} = C^{-1}[\quad]_{B_2}$

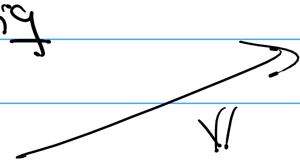
B_1 to B_2 conversion

B_2 to B_1 conversion

Thm 3.3.2 V is a vector space with a basis $v_1, v_2, \ldots, v_n$ (span V, lin. ind)

Given: $v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$

So $\alpha_1, \alpha_2, \ldots, \alpha_n$ are uniq



then every v have unig $\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$ basis v_i

coordinates of v using basis v_i

Notation

$[v]_B$ ← basis you are using

↑ coord. of your vector

Standards

$\mathbb{R}^n$ <u>basis</u> $e_1, e_2, \ldots, e_n$

ex $\mathbb{R}^3$ <u>standard basis</u> $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ← E

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_E = 1e_1 + 2e_2 + 3e_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

standard   standard

P_n standard basis $1, x, x^2, x^3, \ldots, x^{n-1}$

Change between bases

if you have standard basis $e_1, e_2, \ldots, e_n$ and a second basis $u_1, u_2, \ldots, u_n$ → $u_n = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_E$

let $U = [u_1 \ u_2 \ \ldots \ u_n]$

and say that we have the coord. of v in basis U

So $\quad V = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}_u \quad [V]_u$

Consider $\quad U \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}_u = [u_1\ u_2 \cdots u_n] \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}_u$

$$V = c_1 u_1 + c_2 u_2 + \cdots + c_n u_n = c_1 \begin{bmatrix} u_{11} \\ u_{21} \\ \vdots \\ u_{n1} \end{bmatrix}_E + c_2 \begin{bmatrix} u_{12} \\ \vdots \\ u_{n2} \end{bmatrix}_E$$

$$= \begin{bmatrix} c_1 u_{11} + c_2 u_{12} \cdots \\ \\ c_1 u_{n1} + c_2 u_{n2} + \cdots \end{bmatrix}_E$$

So

$$\boxed{\begin{gathered} [v]_E = U [v]_u \\ U = [u_1\ u_2 \cdots u_n] \\ U^{-1} [v]_E = [v]_u \end{gathered}}$$

Ex $\quad u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$V = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}_u \rightarrow V = [v]_E\ ?$

$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad$ So $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}_u = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}_E$