Math 511

Q's of Basis

from Induction: (logical Induction)

four property of 1, 2, 3, 4, 5, ...

you can make a seq of objects for a set ..

a) Basis = given 1^{st} elements of your set

b) Inductive or recursive → make new elements from old elements

for Math 511

Basis → starting elements

set $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$

Inductive part: make new elements

if $\mathbb{V}_1, \mathbb{V}_2$ in our set

then $\mathbb{V}_1 + \mathbb{V}_2$ in our set

$\alpha \mathbb{V}_1$ in our set

$\alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2 + \alpha_3 \mathbb{V}_3 = \mathbb{V}$

the set we make is the $\boxed{\text{Span}(\mathbb{V}_1, \mathbb{V}_2, \mathbb{V}_3)}$

3.5 #c $\quad B_1 = [x, 1] \quad B_2 = [2x-1, 2x+1]$

P_2 $\quad$ use $p(x) = ax+b \rightarrow$ use $\boxed{\begin{bmatrix} a \\ b \end{bmatrix} \text{ for } p}$

#6 $\quad B = [1, 1+x, 1+x+x^2]$

P (any poly) $\quad p(x) = 2+x-x^2+0x^3+0x^4+0x^5+\dots$

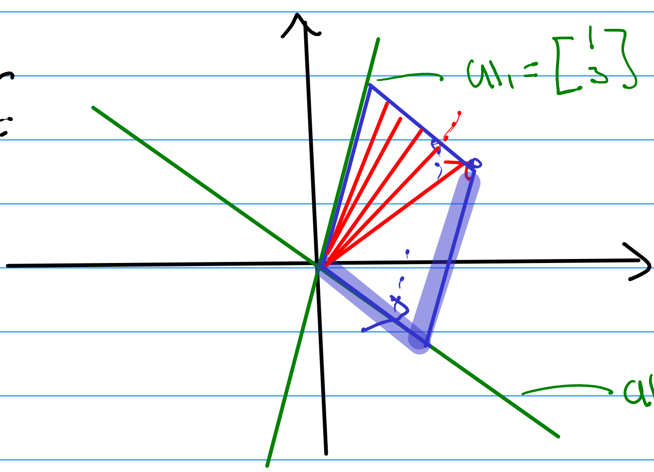
$= 2+x-x^2$

P_4 $\quad p(x) = 0x^3 - x^2 + x + 2$

$p(x) = 2+x-x^2+0x^3$

---

eigen Value/Vector



$a_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad \lambda_1 = 1$ $\quad A v /\!/$

$a_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad \lambda_2 = 0.92$

3.6 $\quad A = [a_1, a_2 \dots a_n] = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix}$ $\quad$ size : $m \times n$

$a_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \in \mathbb{R}^m$

$\vec{a}_1 = [a_{11} a_{12} \cdots a_{1n}]$

$\vec{a}_1 \in \mathbb{R}^{1\times n}$ or $(\mathbb{R}^n)^T$

Consider $A = [a_{1,1}\ a_{1,2} \cdots a_{1,n}] = \begin{bmatrix} \vec{a}_{1,1} \\ \vec{a}_{1,2} \\ \vdots \\ \vec{a}_{1,m} \end{bmatrix}$

$V = \alpha_1 a_{1,1} + \alpha_2 a_{1,2} + \cdots + \alpha_n a_n$ so $V \in \mathbb{R}^m$

$\Rightarrow$ all these linear combos $\mathrm{Span}(a_{1,1}, a_{1,2}, \ldots, a_{1,n})$

Know: $\mathrm{Span}(a_{1,1}, a_{1,2}, \ldots, a_{1,n})$ is a subspace of $\mathbb{R}^m$

Def: $\mathrm{Span}(a_{1,1}, a_{1,2}, \ldots, a_{1,n})$ is called the $\boxed{\text{column space of } A}$

Consider $\vec{V} = \alpha_1 \vec{a}_{1,1} + \alpha_2 \vec{a}_{1,2} + \cdots + \alpha_m \vec{a}_{1,m}$

ith $\mathbb{R}^{1\times n}$

So $\vec{V} \in \mathbb{R}^{1\times n}$

all such $\vec{V}$ is $\mathrm{Span}(\vec{a}_{1,1}, \vec{a}_{1,2}, \ldots, \vec{a}_{1,m})$

know this is a subspace of $\mathbb{R}^{1\times n}$

let's call $\mathrm{Span}(\vec{a}_{1,1}, \vec{a}_{1,2}, \ldots, \vec{a}_{1,m})$ the $\boxed{\text{row space of } A.}$

Ex 3 $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \\ 1 & 5 & 5 \end{bmatrix}$

col space of $A = \mathrm{Span}\left(\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}\right)$

row space of A = Span$([1\ 1\ 2], [-1\ 3\ 1], [1\ 5\ 5])$

1st Study row space of A.

row space of A = { all linear combos of A's rows }

Compare to
$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \\ 1 & 5 & 5 \end{bmatrix} \xrightarrow[\text{ops}]{\text{row}} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 4 & 3 \end{bmatrix} \xrightarrow[\text{ops}]{\text{row}} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \\ 1 & 5 & 5 \end{bmatrix} \xrightarrow[\text{row ops}]{\text{elem.}} U = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

linear combos of A's row vectors.

Thm two row equiv. matrices have the same row space.

(b/c elem row ops are simply linear combos of rows)

So $$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \\ 1 & 5 & 5 \end{bmatrix} \xrightarrow[\text{equiv}]{\text{row}} U = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

row space of A = Span$([1\ 1\ 2], [-1\ 3\ 1], [1\ 5\ 5])$

by thm

= row space of U = Span$([1\ 1\ 2], [0\ 4\ 3])$

$\therefore$ basis of row space of A = $\{[1\ 1\ 2], [0\ 4\ 3]\}$

$\therefore$ dim(row space of A) = 2

Notice: dim(row space of A) = # of lead variables

Def: rank(A) = # of lead vars = dim(row space of A)

2nd Study Column Space

Col Space of A = { any linear combo of A's col's }

Compare: to $A\mathbb{X} = \mathbb{b}$

old thm

$$x_1 a_1 + x_2 a_2 + \cdots + x_n a_n = \mathbb{b}$$

Soln if $\mathbb{b}$ is a linear combo of A's col's.

thm $A\mathbb{X} = \mathbb{b}$ is consistent iff $\mathbb{b}$ is in A's Col. Space.

is a subspace of $\mathbb{R}^n$

① $A\mathbb{X} = \mathbb{b}$ has a soln for all $\mathbb{b} \in \mathbb{R}^n$ iff Col space of A spans $\mathbb{R}^n$.

② $A\mathbb{X} = \mathbb{b}$ has at most one soln for every $\mathbb{b} \in \mathbb{R}^n$ iff A's col's are linearly ind.

Corollary

$A_{n \times n}$ is non-singular iff col. vectors of A form a basis of $\mathbb{R}^n$